INTRODUCTION

- The cycle is defined as the repeated series of operation or processes performed on a system, so that the system attains its original state.
- The cycle which uses air as the working fluid is known as Gas power cycles.
- In the gas power cycles, air in the cylinder may be subjected to a series of operations which causes the air to attain to its original position.
- The source of heat supply and the sink for heat rejection are assumed to be external to the air.
- The cycle can be represented usually on $p-V$ and $T-S$ diagrams.

POWER CYCLES

- Ideal Cycles, Internal Combustion
  - Otto cycle, spark ignition
  - Diesel cycle, compression ignition
  - Sterling & Ericsson cycles
  - Brayton cycles
  - Jet-propulsion cycle
- Ideal Cycles, External Combustion
  - Rankine cycle

IDEAL CYCLES

- Idealizations & Simplifications
  - Cycle does not involve any friction
  - All expansion and compression processes are quasi-equilibrium processes
- Pipes connecting components have no heat loss
- Neglecting changes in kinetic and potential energy (except in nozzles & diffusers)

**GAS POWER CYCLES**
- Working fluid remains a gas for the entire cycle
- Examples:
  - Spark-ignition engines
  - Diesel engines
  - Gas turbines

**Air-Standard Assumptions**
- Air is the working fluid, circulated in a closed loop, is an ideal gas
- All cycles, processes are internally reversible
- Combustion process replaced by heat-addition from external source
- Exhaust is replaced by heat rejection process which restores working fluid to initial state

**ENGINE TERMS**
- Top dead center
- Bottom dead center
- Bore
- Stroke

- Clearance volume
- Displacement volume
CYCLES AND THEIR CONCEPTS

OTTO CYCLE

An Otto cycle is an idealized thermodynamic cycle that describes the functioning of a typical spark ignition piston engine. It is the thermodynamic cycle most commonly found in automobile engines. The idealized diagrams of a four-stroke Otto cycle both diagrams

- Petrol and gas engines are operated on this cycle
- Two reversible isentropic or adiabatic processes
- Two constant volume process
PROCESS OF OTTO CYCLE

- **Ideal** Otto Cycle
- Four internally reversible processes
  - 1-2 Isentropic compression
  - 2-3 Constant-volume heat addition
  - 3-4 Isentropic expansion
  - 4-1 Constant-volume heat rejection

Thermal efficiency of ideal Otto cycle:
Since $V_2 = V_3$ and $V_4 = V_1$

\[ \eta_{\text{th, out}} = 1 - \frac{1}{\rho^{k-1}} \]

**DIESEL CYCLE**

The **Diesel cycle** is a combustion process of a reciprocating internal combustion engine. In it, fuel is ignited by heat generated during the compression of air in the combustion chamber, into which fuel is then injected.

It is assumed to have constant pressure during the initial part of the "combustion" phase.

The Diesel engine is a heat engine: it converts heat into work. During the bottom isentropic processes (blue), energy is transferred into the system in the form of work $W_{\text{in}}$ by definition (isentropic) no energy is transferred into or out of the system in the form of heat. During the constant pressure (red, isobaric) process, energy enters the system as heat $Q_{\text{in}}$.

During the top isentropic processes (yellow), energy is transferred out of the system in the form of $W_{\text{out}}$, but by definition (isentropic) no energy is transferred into or out of the system in the form of heat. During the constant volume (green, isochoric) process, some of energy flows out of the system as heat through the right depressurizing process $Q_{\text{out}}$. The work that leaves the system is equal to the work that enters the system plus the difference between the heat added to the system and the heat that leaves the system; in other words, net gain of work is equal to the difference between the heat added to the system and the heat that leaves the system.

**PROCESSES OF DIESEL CYCLE:**

- 1-2 Isentropic compression
- 2-3 Constant-Pressure heat addition
- 3-4 Isentropic expansion
- 4-1 Constant-volume heat rejection
For ideal diesel cycle

\[
\eta_{\text{th, Diesel}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}
\]

Cut off ratio \( r_c \),

\[
r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}
\]

Efficiency becomes

\[
\eta_{\text{th, Diesel}} = 1 - \frac{1}{x^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]
\]

**DUAL CYCLE**

The dual combustion cycle (also known as the limited pressure or mixed cycle) is a thermal cycle that is a combination of the Otto cycle and the Diesel cycle. Heat is added partly at constant volume and partly at constant pressure, the advantage of which is that more time is available for the fuel to completely combust. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.

- Heat addition takes place at constant volume and constant pressure process.
- Combination of Otto and Diesel cycle.
- Mixed cycle or limited pressure cycle

**PROCESS OF DUAL CYCLE**

![T-S Diagram](image)
- Isentropic compression
- Constant-volume heat rejection
- Constant-pressure heat addition
- Isentropic expansion
- Constant-volume heat rejection

The cycle is the equivalent air cycle for reciprocating high speed compression ignition engines. The P-V and T-s diagrams are shown in Figs. 6 and 7. In the cycle, compression and expansion processes are isentropic; heat addition is partly at constant volume and partly at constant pressure while heat rejection is at constant volume as in the case of the Otto and Diesel cycles.

**BRAYTON CYCLE**

The Brayton cycle is a thermodynamic cycle that describes the workings of a constant pressure heat engine. Gas turbine engines and airbreathing jet engines use the Brayton Cycle. Although the Brayton cycle is usually run as an open system (and indeed must be run as such if internal combustion is used), it is conventionally assumed for the purposes of thermodynamic analysis that the exhaust gases are reused in the intake, enabling analysis as a closed system. The Ericsson cycle is similar to the Brayton cycle but uses external heat and incorporates the use of a regenerator.

- Gas turbine cycle
- Open vs closed system model
With cold-air-standard assumptions

\[ \eta_{th, Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)} \]

Since processes 1-2 and 3-4 are isentropic, \( P_2 = P_3 \) and \( P_4 = P_1 \)

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = \left( \frac{P_3}{P_4} \right)^{(k-1)/k} = \frac{T_3}{T_4} \]

Pressure ratio is

\[ r_p = \frac{P_2}{P_1} \]

Efficiency of Brayton cycle is

\[ \eta_{th, Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}} \]
REAL TIME APPLICATIONS

PETROL ENGINES

- Datsun Go
- Hyundai Xcent
- Maruti Suzuki Celerio
- Volkswagen Vento
- Nissan Terrano

DIESEL ENGINES

- Isuzu Diesel Cars
- Datsun Diesel Cars
- Ashok Leyland Diesel Cars

GAS TURBINES

- Indraprastha (Delhi) CCGT Power Station India
- Kovilkalappal (Thirumakotai) Gas CCGT Power Station India
- Lanco Tanjore (Karuppur) CCGT Power Plant India

TECHNICAL TERMS

- **TDC**: Top Dead Center: Position of the piston where it forms the smallest volume
- **BDC**: Bottom Dead Center: Position of the piston where it forms the largest volume
- **Stroke**: Distance between TDC and BDC
- **Bore**: Diameter of the piston (internal diameter of the cylinder)
- **Clearance volume**: Ratio of maximum volume to minimum volume \( \frac{V_{BDC}}{V_{TDC}} \)
- **Engine displacement**: \((\text{no of cylinders}) \times \text{stroke length} \times \text{bore area})\) (usually given in cc or liters)
- **MEP**: mean effective pressure: A constant theoretical pressure that if acts on piston produces work same as that during an actual cycle
- **Gas Power Cycles**: Working fluid remains in the gaseous state through the cycle. Sometimes useful to study an idealised cycle in which internal irreversibilities and complexities are removed. Such cycles are called: Air Standard Cycles
- **The mean effective pressure (MEP)**: A fictitious pressure that, if it were applied to the piston during the power stroke, would produce the same amount of net work as that produced during the actual cycle.
- **Thermodynamics**: Thermodynamics is the science of the relations between heat, work and the properties of system
- **Boundary**: System is a fixed and identifiable collection of matter enclosed by a real or imaginary surface which is impermeable to matter but which may change its shape or volume. The surface is called the boundary
- **Surroundings**: Everything outside the system which has a direct bearing on the system's behavior.
- **Extensive Property**: Extensive properties are those whose value is the sum of the values for each subdivision of the system, eg mass, volume.
- **Intensive Property:** Properties are those which have a finite value as the size of the system approaches zero, e.g. pressure, temperature, etc.

- **Equilibrium:** A system is in thermodynamic equilibrium if no tendency towards spontaneous change exists within the system. Energy transfers across the system disturb the equilibrium state of the system but may not shift the system significantly from its equilibrium state if carried out at low rates of change. I mentioned earlier that to define the properties of a system, they have to be uniform throughout the system. Therefore to define the state of system, the system must be in equilibrium. Inequilibrium of course implies non-uniformity of one or more properties).

- **Isentropic process:** Isentropic process is one in which for purposes of engineering analysis and calculation, one may assume that the process takes place from initiation to completion without an increase or decrease in the entropy of the system, i.e., the entropy of the system remains constant.

- **Isentropic flow:** An isentropic flow is a flow that is both adiabatic and reversible. That is, no heat is added to the flow, and no energy transformations occur due to friction or dissipative effects. For an isentropic flow of a perfect gas, several relations can be derived to define the pressure, density and temperature along a streamline.

- **Adiabatic heating:** Adiabatic heating occurs when the pressure of a gas is increased from work done on it by its surroundings, e.g. a piston. Diesel engines rely on adiabatic heating during their compression.

- **Adiabatic cooling:** Adiabatic cooling occurs when the pressure of a substance is decreased as it does work on its surroundings. Adiabatic cooling occurs in the Earth's atmosphere with orographic lifting and lee waves. When the pressure applied on a parcel of air decreases, the air in the parcel is allowed to expand; as the volume increases, the temperature falls and internal energy decreases.

### SOLVED PROBLEMS

1. In an Otto cycle air at 1bar and 290K is compressed isentropically until the pressure is 15bar. The heat is added at constant volume until the pressure rises to 40bar. Calculate the air standard efficiency and mean effective pressure for the cycle. Take $C_v=0.717$ KJ/Kg K and $R_{univ} = 8.314$ KJ/Kg K.

**Given Data:**

- Pressure ($P_1$) = 1bar = 100KN/m$^2$
- Temperature($T_1$) = 290K
- Pressure ($P_2$) = 15bar = 1500KN/m$^2$
- Pressure ($P_3$) = 40bar = 4000KN/m$^2$
- $C_v = 0.717$ KJ/KgK
- $R_{univ} = 8.314$ KJ/Kg K
To Find:

i) Air Standard Efficiency ($\eta_{\text{otto}}$)

ii) Mean Effective Pressure ($P_m$)

Solution:

Here it is given $R_{\text{univ}} = 8.314 \text{ KJ/Kg K}$

We know that,

$\gamma = \frac{C_p}{C_v}$ (Here $C_p$ is unknown)

$R_{\text{univ}} = M \times R$

Since For air ($O_2$) molecular weight ($M$) = 28.97

$8.314 = 28.97 \times R$

$\therefore R = 0.2869$

(Since gas constant $R = C_p - C_v$)

$0.2869 = C_p - 0.717$

$\therefore C_p = 1.0039 \text{ KJ/Kg K}$

$$\gamma = \frac{C_p}{C_v} = \frac{1.0039}{0.717} = 1.4$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

Here ‘$r$’ is unknown.

We know that,

$$r = \left(\frac{V_1}{V_2}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$= \left(\frac{1500}{100}\right)^{\frac{1}{1.4}}$$

$$r = 6.919$$

$$\eta_{\text{otto}} = 1 - \frac{1}{6.919^{0.4}}$$
\[ \therefore \eta_{\text{ Otto }} = 3.87\% \]

Mean Effective Pressure (\( P_m \)):

\[
P_m = P_1 \left( \frac{1}{1 - \frac{(r-1)(\gamma - 1)}{(\gamma - 1)}} \right)
\]

\[
P_m = \frac{(100)(6.919)(2.67 - 1)(6.919^{\frac{r-1}{(\gamma - 1)}} - 1)}{(\gamma - 1)(6.919 - 1)}
\]

\[ P_m = 569.92 \, \text{KN/m}^2 \]

2. Estimate the lose in air standard efficiency for the diesel engine for the compression ratio 14 and the cutoff changes from 6% to 13% of the stroke.

**Given Data**

<table>
<thead>
<tr>
<th>C</th>
<th>Case (i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression ratio (( r )) = 14</td>
<td>compression ratio (( r )) = 14</td>
</tr>
<tr>
<td>( \rho = 6% , V_s )</td>
<td>( \rho = 13% , V_s )</td>
</tr>
</tbody>
</table>

**To Find**

Lose in air standard efficiency.

**Solution**

Compression ratio (\( r \)):

\[
r = \frac{V_1}{V_2} = \frac{V_c + V_s}{V_c}
\]

\[ 14 = 1 + \frac{V_s}{V_c} \]

\[
\frac{V_c}{V_s} = 13
\]

**Case (i):**

Cutoff ratio (\( \rho \)) = \( \frac{V_3}{V_2} \)

\[
\frac{V_3}{V_2} = \frac{V_c + 6\% V_s}{V_c}
\]
\[ \rho = 1 + \frac{6\% V_s}{V_c} \]

\[ \rho = \frac{V_3}{V_2} = 1 + (0.06)(13) \]

\[ \rho = 1.78 \]

We know that,

\[ \eta_{\text{diesel}} = 1 - \frac{1}{\gamma \cdot e^{\frac{1}{\rho - 1}}} \cdot \left[ \frac{\rho^\gamma - 1}{\rho - 1} \right] \]

\[ = 1 - \left( \frac{1}{(1.4)(1.4)^{1.54}} \right) \cdot \frac{1.78^{1.54} - 1}{1.78 - 1} \]

\[ = 1 - (0.2485)(1.5919) \]

\[ = 0.6043 \times 100\% \]

\[ \eta_{\text{diesel}} = 60.43\% \]

**case (ii):**

Cutoff ratio (\(\rho\))

\[ \frac{V_3}{V_2} = \frac{V_2 + 13\% V_3}{V_2} \]

\[ = 1 + (0.13) (13) \]

\[ \rho = 2.69 \]

\[ \eta_{\text{diesel}} = 1 - \frac{1}{\gamma \cdot e^{\frac{1}{\rho - 1}}} \cdot \left[ \frac{\rho^\gamma - 1}{\rho - 1} \right] \]

\[ = 1 - \left( \frac{1}{(1.4)(1.4)^{1.54}} \right) \cdot \frac{2.69^{1.54} - 1}{2.69 - 1} \]

\[ = 1 - (0.2485)(1.7729) \]

\[ = 0.5593 \times 100\% \]

\[ = 55.93\% \]

Lose in air standard efficiency = (\(\eta_{\text{diesel case(i)}}\) - (\(\eta_{\text{diesel case(ii)}}\))

\[ = 0.6043 - 0.5593 \]
3. The compression ratio of an air standard dual cycle is 12 and the maximum pressure on the cycle is limited to 70 bar. The pressure and temperature of the cycle at the beginning of compression process are 1bar and 300K. Calculate the thermal efficiency and Mean Effective Pressure. Assume cylinder bore = 250 mm, Stroke length = 300 mm, Cp=1.005 KJ/Kg K, Cv=0.718 KJ/Kg K.

**Given data:**
- Assume $Q_{s1} = Q_{s2}$
- Compression ratio ($r$) = 12
- Maximum pressure ($P_3$) = ($P_4$) = 7000 KN/m$^2$
- Temperature ($T_1$) = 300K
- Diameter (d) = 0.25m Stroke length (l) = 0.3m

**To find:**
- Dual cycle efficiency ($\eta_{dual}$)
- Mean Effective Pressure ($P_m$)

**Solution:**

By Process 1-2:

$$\frac{T_2}{T_1} = \left[\frac{V_2}{V_1}\right]^{\gamma-1} = [r]^{\gamma-1}$$

$$T_2 = 300[12]^{1+\frac{2}{\gamma}}$$

$$T_2 = 810.58K$$

$$\frac{P_2}{P_1} = \left[\frac{V_1}{V_2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$P_2 = [12]^{1.4} \times 100$$

$$P_2 = 3242.3 \text{ KN/m}^2$$
By process 2-3:

\[
\begin{align*}
  \frac{P_2}{T_2} &= \frac{P_3}{T_3}, \\
  \frac{P_3}{T_3} &= \frac{P_2}{T_2}.
\end{align*}
\]

Assuming \( Q_{s1} = Q_{s2} \)

\[ T_3 = \left[ \frac{7000}{3242.3} \right] 810.58 \]

\[ T_3 = 1750K \]

By process 4-5:

\[ mCv[T_3-T_2] = mCp[T_4-T_3] \cdot 0.718 \]

\[ [1750-810.58] = 1.005 [T_4-1750] \]

\[ T_4 = 2421.15K \]

\[ \frac{T_4}{T_5} = \left[ \frac{V_5}{V_4} \right]^{\gamma - 1} \]

We know that,

\[ \frac{T_4}{T_5} = \left[ \frac{V_4}{V_3} \right] - \frac{1}{\rho} \]

\[ \rho = \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{2421.15}{1750} = 1.38 \]

\[ \frac{T_4}{T_5} = \left[ \frac{12}{1.38} \right]^{\gamma - 1} \]

\[ T_5 = \frac{2421.15}{\left( \frac{12}{1.38} \right)^{0.4}} \]

\[ T_5 = 1019.3K \]
Heat supplied

\[ Q_s = 2 \times m \times C_v \times [T_3 - T_2] \]

\[ = 2 \times 1 \times 0.718 \times [1750 - 810.58] \]

\[ Q_s = 1349 \text{KJ/Kg} \]

Heat rejected

\[ Q_r = m \times C_v \times [T_5 - T_1] \]

\[ = 516.45 \text{ KJ/Kg} \]

\[ \eta_{\text{dual}} = \frac{Q_s - Q_r}{Q_s} \times 100 \]

\[ \eta_{\text{dual}} = 61.72\% \]

Stroke volume

\[ (V_s) = \frac{\pi}{4} \times d^2 \times l \]

\[ = \frac{\pi}{4} \times 0.25^2 \times 0.3 \]

\[ V_s = 0.0147 \text{m}^3 \]

Mean Effective Pressure (Pm)

\[ = \frac{W}{V_s} \]

\[ = 832.58/0.0147 \]

\[ P_m = 56535 \text{ KN/m}^2 \]

4. A diesel engine operating an air standard diesel cycle has 20cm bore and 30cm stroke. The clearance volume is 420cm³. If the fuel is injected at 5% of the stroke, find the air standard efficiency.

**Given Data:**

Bore diameter (d) = 20cm = 0.2mk

Stroke, (l) = 30cm = 0.3m

Clearance volume, \( (v_2) = 420 \text{cm}^3 = 420/100^3 = 4.2 \times 10^{-4} \text{m}^3 \)

**To Find:**-
Air standard efficiency, (diesel) Solution:-
Compression ratio, 
\[ r = \frac{v_1}{v_2} \]
We know that,

Stroke volume, 
\[ v_s = \text{area} \times \text{length} \]
\[ = \left( \frac{\pi}{4} \right) d^2 \times l \]
\[ = \left( \frac{\pi}{4} \right) (0.2^2) \times 0.3 \]
\[ V_s = 9.4 \times 10^{-3} \text{ m}^3 \]
Therefore,

Compression ratio, 
\[ (r) = \frac{4.2 \times 10^{-4} + 9.4 \times 10^{-3}}{4.2 \times 10^{-4}} \]
\[ r = 23.42 \]
Cut off ratio, 
\[ \rho = \frac{v_3}{v_2} + 5\% \]
\[ = \frac{(v_2 + 5\% v_2)}{v_2} \]
\[ = 1 + \frac{(0.05 \times 9.4 \times 10^{-3})}{4.2 \times 10^{-4}} \]
\[ \rho = 2.12 \]
We know the equation,
\[ \eta_{diesel} = 1 - \frac{1}{\gamma(r)\gamma-1} \times \left( \frac{\rho^\gamma - 1}{\rho - 1} \right) \]
\[ = 1 - \frac{1}{14 \times 29.42^4 + 1} \times \left( \frac{2.12^{2.4} - 1}{2.12 - 1} \right) \]
\begin{align*}
\eta_{\text{diesel}} &= 1 - (0.20229)(1.6636) \\
&= 0.6634 \times 100 \\
\eta_{\text{diesel}} &= 66.34\% 
\end{align*}
UNIT II RECIPROCATING AIR COMPRESSOR

Introduction:
- The process of increasing the pressure of air, gas or vapour by reducing its volume is what compression.
- The devise used to carry out this process is called a compressor.

Principles on which compressors work:
- A compressor is a mechanical device that increases the pressure of a gas by reducing its volume.
- Compressor is a machine which increases the pressure of a fluid by mechanically decreasing its volume (i.e. by compressing it).(The fluid here is generally air since liquids are theoretically incompressible).

Construction:
Compressed Air is often described as the fourth utility, although not as ubiquitous as electricity, petrol and gas, it plays a fundamental part in the modern world.

The importance of compressed air is often overlooked, but in reality it plays a vital part in most modern manufacturing processes and modern civilization.

Although we may not realize it most products we use today could simply not be made without compressed air.

Compressed air accounts for about 10% of the global energy used in industry today.

With so many applications in different environments being dependant on compressed air, the compressors not only have to compress the air to a specific pressure, at a certain flow, it has to deliver air of the right quality.

To most people, a compressor is all that is required to compress air, but to obtain the right quality of the compressed air, more equipment is often needed.

Filters and dryers are often needed to remove oil and water before it reaches the application.

Compressed Air has a range of completely oil-less compressors where air comes into contact with the process it serves and so the quality is critical, for example in where a compressor may be used in a food packaging role.

CLASSIFICATION OF AIR COMPRESSORS

Types of compressors
Positive displacement compressor

- In the positive-displacement type, a given quantity of air or gas is trapped in a compression chamber and the volume it occupies is mechanically reduced, causing a corresponding rise in pressure prior to discharge.

- At constant speed, the air flow remains essentially constant with variations in discharge pressure.

- Ex: Reciprocating compressors, vane compressors & so on.

Dynamic compressors:

- Dynamic compressors impart velocity energy to continuously flowing air or gas by means of impellers rotating at very high speeds.

- The velocity energy is changed into pressure energy both by the impellers and the discharge volutes or diffusers.

- In the centrifugal-type dynamic compressors, the shape of the impeller blades determines the relationship between air flow and the pressure (or head) generate.

- Ex: centrifugal compressors, axial compressors.

Reciprocating compressors

- In a reciprocating compressor, a volume of gas is drawn into a cylinder; it is trapped and compressed by piston, then discharged into the discharge line.

- The cylinder valves control the flow of gas through the cylinder; these valves act as check valves.
Principle of Operation

- The piston is driven by a crank shaft via a connecting rod.
- At the top of the cylinder are a suction valve and a discharge valve.
- A reciprocating compressor usually has two, three, four, or six cylinders in it.

- The suction valve opens at point 4.
- As the piston travels toward the bottom dead center, the volume of the cylinder increases and the vapor flows into the cylinder.
- The pressure inside the cylinder is slightly less than suction line pressure. The pressure difference pushes the valve open on during the suction stroke.
At point 2, the pressure inside the cylinder has become slightly greater than discharge line pressure.

This causes the valve opening allowing the gas to flow out of the cylinder.

The volume continues to decrease toward point 3, maintaining a sufficient pressure difference across the discharge valve to hold it open.

At point 3, the piston reaches the top dead center and reverses direction.

At top dead center, as the piston comes to a complete stop prior to reversing direction, the pressure across the valve is equal.

So, the discharge valve is closed.

As the piston moves towards point 4, the volume increases and the pressure decreases in the cylinder.

The gas trapped in the cylinder expands as the volume increases until to point 4.

At point 4, the gas pressure inside the cylinder becomes less than the suction line pressure, so the suction valve opens again.

The cycle then starts over again.

The shape of the re-expansion line (Line 3-4) is dependent on the same compression exponent that determines the shape of the compression line.

**What is the difference between a single and two stage compressor?**

The simplest way to explain the difference between a single stage compressor and dual or two stage compressor is the number of times that the air is compressed. In a single stage system the air is compressed once and in a dual stage the air is compressed twice.

In a single stage piston compressor the air is drawn into a cylinder and compressed in a single piston stoke to a pressure of approximately 120 PSI. Then it is send to the storage tank. All rotary compressors are single stage.

In a dual stage compressor the first step is the same except that the air is not directed to the storage tank, the air is sent via an inter cooler tube to a second, smaller high pressure piston and compressed a second time and compressed to a pressure of 175 PSI. Then it is sent through the after cooler to the storage tank.

In a dual stage pump the first stage cylinder is always a larger diameter. Also a dual stage pump will always have an inter cooler tube or finned housing attached to the pump to cool the air before being compressed a second time.

**ROTARY VANE COMPRESSORS**

**Rotary vane compressors** consist of a rotor with a number of blades inserted in radial slots in the rotor.
The rotor is mounted offset in a larger housing that is either circular or a more complex shape. As the rotor turns, blades slide in and out of the slots keeping contact with the outer wall of the housing. Thus, a series of decreasing volumes is created by the rotating blades.

MULTISTAGE COMPRESSION:
Multistage compression refers to the compression process completed in more than one stage i.e., a part of compression occurs in one cylinder and subsequently compressed air is sent to subsequent cylinders for further compression. In case it is desired to increase the compression ratio of compressor then multi-stage compression becomes inevitable. If we look at the expression for volumetric efficiency then it shows that the volumetric efficiency decreases with increase in pressure ratio. This aspect can also be explained using p-V representation shown in Figure.

A multi-stage compressor is one in which there are several cylinders of different diameters. The intake of air in the first stage gets compressed and then it is passed over a cooler to achieve a temperature very close to ambient air. This cooled air is passed to the intermediate stage where it is again getting compressed and heated. This air is again passed over a cooler to achieve a temperature as close to ambient as possible. Then this compressed air is passed to the final or the third stage of the air compressor where it is compressed to the required pressure and delivered to the air receiver after cooling sufficiently in an after-cooler.
Advantages of Multi-stage compression:

1. The work done in compressing the air is reduced, thus power can be saved
2. Prevents mechanical problems as the air temperature is controlled
3. The suction and delivery valves remain in cleaner condition as the temperature and vaporization of lubricating oil is less
4. The machine is smaller and better balanced
5. Effects from moisture can be handled better, by draining at each stage
6. Compression approaches near isothermal
7. Compression ratio at each stage is lower when compared to a single-stage machine

WORK DONE IN A SINGLE STAGE RECIPROCATING COMPRESSOR WITH CLEARANCE VOLUME:

Considering clearance volume: With clearance volume the cycle is represented on Figure. The work done for compression of air polytropically can be given by the area enclosed in cycle 1-2-3-4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.
In the cylinder of reciprocating compressor (V1 - V4) shall be the actual volume of air delivered per cycle. $V_d = V_1 - V_4$. This $(V_1 - V_4)$ is actually the volume of air inhaled in the cycle and delivered subsequently.

If air is considered to behave as perfect gas then pressure, temperature, volume and mass can be interrelated using perfect gas equation. The mass at state 1 may be given as $m_1$ mass at state 2 shall be $m_1$, but at state 3 after delivery mass reduces to $m_2$ and at state 4 it shall be $m_2$.

$$W_{c, with CV} = \frac{n}{n-1} p_1 V_t \left( \frac{p_2}{p_1} \right)^{\frac{n}{n-1}} - 1$$

Here $p_1 = p_4$, $p_2 = p_3$

$$= \left( \frac{n}{n-1} \right) \left( p_1 V_t \right) \left( \frac{p_2}{p_1} \right)^{\frac{n}{n-1}} - 1 - \left( \frac{n}{n-1} \right) \left( p_3 V_4 \right) \left( \frac{p_2}{p_3} \right)^{\frac{n}{n-1}} - 1$$

$$= \left( \frac{n}{n-1} \right) \left( p_1 \right) \left( \frac{p_2}{p_1} \right)^{\frac{n}{n-1}} - 1 \left( V_t - V_4 \right)$$

Ideally there shall be no change in temperature during suction and delivery i.e., $T_4 = T_1$ and $T_2 = T_3$ from earlier.
Thus \((m_1 - m_2)\) denotes the mass of air sucked or delivered. For unit mass of air delivered the work done per kg of air can be given as,

\[
W_{c, \text{with CV}} = \left( \frac{n}{n-1} \right) m_i R \left( T_1 - T_i \right) \left( \frac{T_1}{T_i} - 1 \right)
\]

Substituting

\[
W_{c, \text{with CV}} = \left( \frac{n}{n-1} \right) m_i R T_1 - m_i R T_i \left( \frac{T_i}{T_1} - 1 \right)
\]

Substituting for constancy of temperature during suction and deliver

\[
W_{c, \text{with CV}} = \left( \frac{n}{n-1} \right) m_i R T_1 \left( \frac{T_i}{T_1} - 1 \right)
\]

Or

\[
W_{c, \text{with CV}} = \left( \frac{n}{n-1} \right) (m_i - m_j) R (T_2 - T_1)
\]

Thus from above expressions it is obvious that the clearance volume reduces the effective swept volume i.e., the mass of air handled but the work done per kg of air delivered remains unaffected. From the cycle work estimated as above the theoretical power required for running compressor shall be, For single acting compressor running with \(N\) rpm, power input required, assuming clearance volume.

\[
\text{Power required} = \left( \frac{n}{n-1} \right) R \left( \frac{T_2}{T_i} - 1 \right) \left( \frac{p_2}{p_1} \right) \left( \frac{V_1}{V_i} - 1 \right)
\]

For double acting compressor, Power

\[
\text{Power required} = \left( \frac{n}{n-1} \right) R \left( \frac{T_2}{T_i} - 1 \right) \left( \frac{p_2}{p_1} \right) \left( \frac{V_1}{V_i} - 1 \right) \left( \frac{2}{N} \right)
\]
4.8 VOLUMETRIC EFFICIENCY:

Volumetric efficiency of compressor is the measure of the deviation from volume handling capacity of compressor. Mathematically, the volumetric efficiency is given by the ratio of actual volume of air sucked and swept volume of cylinder. Ideally the volume of air sucked should be equal to the swept volume of cylinder, but it is not so in actual case. Practically the volumetric efficiency lies between 60 to 90%. Volumetric efficiency can be overall volumetric efficiency and absolute volumetric efficiency as given below.

Overall volumetric efficiency = \( \frac{\text{Volume of free air sucked in cylinder}}{\text{Swept volume of LP cylinder}} \)

\( \text{(Volumetric efficiency)}_{\text{free air condition}} = \frac{\text{Volume of free air sucked in cylinder}}{(\text{Swept volume of LP cylinder})_{\text{free air condition}}} \)

Here free air condition refers to the standard conditions. Free air condition may be taken as 1 atm or 1.01325 bar and 15oC or 288K. Consideration for free air is necessary as otherwise the different compressors can not be compared using volumetric efficiency because specific volume or density of air varies with altitude. It may be seen that a compressor at datum level (sea level) shall deliver large mass than the same compressor at high altitude. This concept is used for giving the capacity of compressor in terms of „free air delivery”(FAD). “Free air delivery is the volume of air delivered being reduced to free air conditions”. In case of air the free air delivery can be obtained using perfect gas equation as,

\[ \frac{p_a V_a}{T_a} = \frac{p_i(V_i - V_4)}{T_1} = \frac{p_i(V_2 - V_3)}{T_2} \]

Where subscript a or pa, Va, Ta denote properties at free air conditions

\[ V_a = \frac{p_T}{p_a} \frac{p_i(V_i - V_4)}{T_1} = \text{FAD per cycle} \]

This volume Va gives „free air delivered” per cycle by the compressor. Absolute volumetric efficiency can be defined, using NTP conditions in place of free air conditions.
Here $V_s$ is the swept volume $= V_1 - V_3$ and $V_c$ is the clearance volume $= V_3$

$\eta_{vol} = \frac{FAD}{\text{Swept volume}} = \frac{V_1}{(V_1 - V_3)} = \frac{p_1 T_1 (V_1 - V_3)}{p_o T_o (V_1 - V_3)}$

$\eta_{vol} = \left(\frac{p_1 T_1}{p_o T_o}\right) \left[1 + \left(\frac{V_1}{V_o} - \frac{V_3}{V_o}\right)\right]$  

Here $\frac{V_1}{V_o} = \frac{V_3}{V_o} = \frac{V_1}{V_3}$

$\eta_{vol} = \left(\frac{p_1 T_1}{p_o T_o}\right) \left[1 + \left(\frac{V_1}{V_o}\right) - \left(\frac{V_3}{V_o}\right)\right]$  

$\eta_{vol} = \left(\frac{p_1 T_1}{p_o T_o}\right) \left[1 + C - C \left(\frac{V_1}{V_o}\right)\right]$  

Volumetric efficiency depends on ambient pressure and temperature, suction pressure and temperature, ratio of clearance to swept volume, and pressure limits. Volumetric efficiency increases with decrease in pressure ratio in compressor.

**Mathematical analysis of multistage compressor is done with following assumptions:**

(i) Compression in all the stages is done following same index of compression and there is no pressure drop in suction and delivery pressures in each stage. Suction and delivery pressure remains constant in the stages.

(ii) There is perfect inter cooling between compression stages.

(iii) Mass handled in different stages is same i.e., mass of air in LP and HP stages are same.

(iv) Air behaves as perfect gas during compression.

From combined p-V diagram the compressor work requirement can be given as,
Work requirement in LP cylinder, \( W_{LP} = \left( \frac{n}{n-1} \right) P_i V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \)

Work requirement in HP cylinder, \( W_{HP} = \left( \frac{n}{n-1} \right) P_1 V_2 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \)

For perfect intercooling, \( p_1 V_1 = p_2 V_2 \); and

\[ W_{HP} = \left( \frac{n}{n-1} \right) P_1 V_2 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \]

Therefore, total work requirement, \( W_c = W_{LP} + W_{HP} \), for perfect intercooling

\[ W_c = \left( \frac{n}{n-1} \right) P_i V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + P_i V_2 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \]

\[ = \left( \frac{n}{n-1} \right) P_i V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] + P_i V_2 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \]

Minimum work required in two stage compressor:

Minimum work required in two stage compressor can be given by

\[ W_{c, min} = \left( \frac{n}{n-1} \right) P_i V_1 \cdot 2 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \]

For \( I \) number of stages, minimum work,

\[ W_{c, min} = I \left( \frac{n}{n-1} \right) P_i V_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \]
4.9 APPLICATION OF COMPRESSORS

- Reciprocating compressors are typically used where high compression ratios (ratio of discharge to suction pressures) are required per stage without high flow rates, and the process fluid is relatively dry.

- Screw compressors: Trailer mounted diesel powered units are often seen at construction sites, and are used to power air operated construction machinery.

- P.E.T bottling industries, gas filling stations usually use reciprocating compressors.

- Processing equipment, Oxygen Generators Oil Atomization use compressors of required capacity.

- Air compressors: a compressor that takes in air at atmospheric pressure and delivers it at a higher pressure.

- Compressors serve the basic necessities & form an integral part of the company.

- Pneumatic brakes
- Pneumatic drills
- Pneumatic jacks
- Pneumatic lifts
- Spray painting
- Shop cleaning
- Injecting fuel in Diesel engines
- Refrigeration and Air conditioning systems.

TECHNICAL TERMS

After cooler - Heat exchangers for cooling air or gas discharged from compressors. They provide the most effective means of removing moisture from compressed air and gases.

Air-Cooled Compressors - Air-cooled compressors are machines cooled by atmospheric air circulated around the cylinders or casings.

Base Plate - A metallic structure on which a compressor or other machine is mounted.

Capacity - The capacity of a compressor is the full rated volume of flow of gas compressed and delivered at conditions of total temperature, total pressure, and composition, prevailing at the compressor inlet. It sometimes means actual flow rate, rather than rated volume of flow.

Capacity, Actual - Quantity of gas actually compressed and delivered to the discharge system at rated speed of the machine and under rated pressure conditions. Actual capacity is usually expressed in cubic feet per minute (cfm) at that stage inlet gas conditions.
**Casing** - The pressure containing stationary element that encloses the rotor and associated internal components of a compressor. Includes integral inlet and discharge corrections (nozzles).

**Check valve** - A check valve is a valve that permits flow in one direction only.

**Clearance** - The maximum cylinder volume on a working side of the piston minus the piston displacement volume per stroke. It is usually expressed as a percentage of the displaced volume.

**Clearance Pocket** - An auxiliary volume that may be opened to the clearance space to increase the clearance, usually temporarily, to reduce the volumetric efficiency of the air compressor.

**Compressibility** - A factor expressing the deviation of a gas from the laws of hydraulics.

**Compression, Adiabatic** - This type of compression is effected when no heat is transferred to or from the gas during the compression process.

**Compression, Isothermal** - Isothermal compression is a compression in which the temperature of the gas remains constant. For perfect gases, it is represented by the equation PV is a constant, if the process is reversible.

**Compression, Polytropic** - Compression in which the relationship between the pre-sum and the volume is expressed by the equation PV is a constant.

**Compression Ratio** - The ratio of the absolute discharge; press = to the absolute inlet pressure.

**Critical Pressure** - The limiting value of saturation pressure as the saturation temperature approaches the critical temperature.

**Critical Temperature** - The highest temperature at which well defined liquid and vapor states exist. It is sometimes defined as the highest temperature at which it is possible to liquify a gas by pressure alone.

**Diaphragm** - A stationary element between the stages of a multistage centrifugal compressor. It may include guide vanes for directing the flowing medium to the impeller of the succeeding stage. In conjunction with an adjacent diaphragm, it forms the diffuser surrounding the impeller.

**Diaphragm Routing** - A method of removing heat from the flowing medium by circulation of a coolant in passages built into the diaphragm.

**Diffuser** - A stationary passage surrounding an impeller, in which velocity pressure imparted to the flowing medium by the impeller is converted into static pressure.

**Displacement** - Displacement of a compressor is the piston volume swept out per unit time; it is usually expressed in cubic feet per minute.

**Dynamic Type Compressors** - Machines in which air or gas is compressed by the mechanical action of routing vanes or impellers imparting velocity and pressure to the flowing medium.

**Efficiency** - Any reference to efficiency of a dynamic type compressor must be accompanied by a qualifying statement which identifies the efficiency under consideration, as in the following definitions.
Efficiency, Compression - Ratio of calculated isentropic work requirement to actual thermodynamic work requirement within the cylinder, the Inner as determined from the cylinder indicator card.

Efficiency, Isothermal - Ratio of the work calculated on an isothermal basis to the actual work transferred to the gas during compression.

Efficiency, Mechanical - Ratio of thermodynamic work requirement in the cylinder (a shown by die indicator card) to actual brake horsepower requirement.

Efficiency, Polytropic - Ratio of the polytropic compression energy transferred to the gas no the actual energy transferred to the gas.

Efficiency, Volumetric - Ratio of actual capacity to piston displacement, stated as a percentage.

Exhauster - This is a term sometimes applied to a compressor in which the inlet pressure is less than atmospheric pressure.

Expanders - Turbines or engines in which a gas expands, doing work, and undergoing a drop in temperature. Use of the term usually implies that the drop in temperature is the principle objective. The orifice in a refrigeration system also performs this function, but the expander performs it nearly isentropically, and is thus more effective in cryogenic systems.

Filters - Filters are devices for separating and removing dust and dirt from air before it enters a compressor.

Flange Connection - The flange connection (inlet or discharge) is a means of connecting the casing to the inlet or discharge piping by means of bolted rims (flanges).

Fluidics - The general subject of instruments and controls dependent upon low rate flow of air or gas at low pressure as the operating medium. These usually have no moving parts.

Free Air - Air at atmospheric conditions at any specific location. Because the altitude, barometer, and temperature may vary at different localities and at different times, it follows that this term does not mean air under identical or standard conditions.

Gas - While from a physical point of view a gas is one of the three basic phases of matter, and thus air is a gas, a special meaning is assigned in pneumatics practice. The term gas refers to any gas other than air.

Gas Bearings - Gas bearings are load carrying machine elements permitting some degree of motion in which the lubricant is air or some other gas.

Volumetric Efficiency of the Compressor - It is the ratio of actual volume of air drawn in the compressor to the stroke volume of the compressor.

Mechanical efficiency - It is the ratio of indicated power to shaft power or brake power of motor.

Isentropic efficiency - It is the ratio of the isentropic power to the brake power required to drive the compressor.
Centrifugal compressor - The flow of air is perpendicular to the axis of compressor

Axial flow compressor - The flow of air is parallel to the axis of compressor

Compression - The process of increasing the pressure of air, gas and vapour by reducing its volume is called as compression.

Single acting compressor - The suction, compression and the delivery of air takes on the one side of piston

Double acting compressor - The suction, compression and the delivery of air takes place on both sides of the piston.

Multi stage compressor - The compression of air from initial pressure to the final pressure is carried out in more than one cylinder.

Application of compressed air - Pneumatic brakes, drills, jacks, lifts, spray of paintings, shop cleaning, injecting the fuel in diesel engine, supercharging, refrigeration and in air conditioning systems.

Inter cooler - It is a simple heat exchanger, exchanges the heat of compressed air from low pressure compressor to circulating water before the air enters to high pressure compressor. The purpose of intercooling is to minimize the work of compression.

Isentropic efficiency - It is the ratio of isentropic power to the brake power required to drive the compressor.

Clearance ratio - It is the ratio of clearance volume to the swept volume or stroke volume is called as clearance ratio.

Isothermal efficiency - It is the ratio between isothermal work to the actual work of the compressor.

Compression ratio - The ratio between total volume and the clearance volume of the cylinder is called compression ratio.

Perfect intercooling - When the temperature of the air leaving the intercooler is equal to the original atmospheric air temperature, then the intercooler is called perfect intercooling.

SOLVED PROBLEMS

1. A single stage double acting air compressor of 150KW power takes air in at 16 bar & delivers at 6 bar. The compression follows the law \( PV^{1.35} = C \). the compressor runs at 160rpm with average piston speed of 150 m/min. Determine the size of the cylinder.

Given data

\[ \text{Power (P)} = 150\text{KW} \]
Piston speed \((2\text{IN})\) = 150m/min \(= \frac{\text{150}}{60} = 2.5\frac{\text{m}}{\text{s}}\)

Speed \((N)\) = 160rpm \(= \frac{160}{60} = 2.7\text{rps}\)

Pressure \((P_1)\) = 1bar = 100KN/m\(^2\)

Pressure \((P_2)\) = 6bar = 600KN/m\(^2\)

\(PV^{1.35} = C, \quad n = 1.35\)

Hence it is a polytropic process.

To find

Size of the cylinder \((d)\)?

Solution

It is given that,

\[2\text{IN} = 2.5\text{m/s}\]

\[l = \frac{\text{2IN}}{2\times2.7}\]

\[l = 0.4629\text{m}\]

since \(V_1 = V_S = \frac{\pi}{4}d^2l\)

\[V_1 = V_S = \frac{\pi}{4}d^2(1.4629)\]

\[V_1 = 0.3635d^2\]

We know that,

Power \((P) = 2\times W\times N\) (for double acting) For polytropic process, work done \((W)\) is

\[W = \frac{n}{n-1} \left(P_1 V_1 \right) \left[ \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]\]

\[W = \frac{1.35}{1.35-1} \left(100 \times 0.3635d^2\right) \left[ \left(6\right)^{\frac{1.35-1}{1.35}} - 1\right]\]

\[W = 82.899\]

\[d^2\]

Power \((P) = \frac{2\times W\times N}{2\times W\times N}\)
\[ 150 = 2 \times 82.899 \, d^2 \times 2.7 \]
\[ d^2 = 0.3350 \]
\[ d = 0.57M \]

2. A single stage single acting reciprocating air compressor is required to handle \(30\, m^3\) of free air per hour measured at 1 bar. The delivery pressure is 6.5 bar and the speed is 450 r.p.m allowing volumetric efficiency of 75\%; an isothermal efficiency of 76\% and mechanical efficiency of 80\% Find the indicated mean effective pressure and the power required the compressor.

**Given data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume Pressure</td>
<td>(V_1 = 30, m^2)</td>
</tr>
<tr>
<td>Pressure (P_2)</td>
<td>6.5 bar</td>
</tr>
<tr>
<td>Speed (N)</td>
<td>450 r.p.m</td>
</tr>
<tr>
<td>Volumetric efficiency (\eta_v)</td>
<td>75%</td>
</tr>
<tr>
<td>Isothermal efficiency (\eta_i)</td>
<td>76%</td>
</tr>
<tr>
<td>Mechanical efficiency (\eta_m)</td>
<td>80%</td>
</tr>
</tbody>
</table>

**To find**

The indicated mean effective pressure
The power required to drive the compressor

**Solution**

**Indicated Mean Effective Pressure**

We know that isothermal work done

\[ W_{isothermal} = 2.3 \times V_1 \, P_1 \, \log \left( \frac{P_2}{P_1} \right) \]

\[ = 2.3 \times 10^5 \times 30 \, \log \left( \frac{6.5}{1} \right) \]

\[ = 5609 \times 10^3 \, J/h \]

And indicated work done

\[ = 7380 \, KJ/h \]
We know that swept volume of the piston

\[ V_s = \frac{\text{volume of free air}}{\text{volumetric efficiency}} \]

\[ = \frac{30}{75} \]

\[ = 40\text{m}^3/\text{h} \]

Indicated mean effective pressure

\[ p_m = \frac{\text{indicated work done}}{\text{swept volume}} \]

\[ = 184.5\text{kJ/m}^3 \]

\[ = 184.5\text{KN/m}^2 \]

The power required to drive the compressor We

know that work done by the compressor = \[ \frac{\text{indicated work done}}{\text{mechanical efficiency}} \]

\[ = \frac{7380}{.8} \]

\[ = 9225\text{KJ/h} \]

Therefore the power required to drive the compressor = \[ \frac{9225}{3600} \]

\[ = 2.56\text{KW} \]

Result

Indicated mean effective pressure \( p_m = 184.5\text{KN/m}^2 \)

The power required to drive the compressor = 2.56KW

3. A two stages, single acting air compressor compresses air to 20bar. The air enters the L.P cylinder at 1bar and 27\(^\circ\)c and leaves it at 4.7bar. The air enters the H.P. cylinder at 4.5bar and 27\(^\circ\)c. the size of the L.P cylinder is 400mm diameter and 500mm stroke. The clearance volume In both cylinder is 4% of the respective stroke volume. The compressor runs at 200rpm, taking

index of compression and expansion in the two cylinders as 1.3, estimate 1. The indicated power required to run the compressor; and 2. The heat rejected in the intercooler per minute.
Given data

Pressure (P4) = 20bar
Pressure (P1) = 1bar = 1×10^5 N/m^2
Temperature (T1) = 27°C = 27+273 = 300K
Pressure (P2) = 4.7bar
Pressure (P3) = 4.5bar
Temperature (T3) = 27°C = 27+273 = 300K
Diameter (D1) = 400mm 0.4m Stroke (L1) = 500mm = 0.5m

\[ K = \frac{v_{x1}}{v_{x1}} = \frac{v_{x3}}{v_{x3}} = \frac{4}{9} = 0.04 \]

N = 200rpm ; n = 1.3

To find

Indicated power required to run the compressor

Solution

We know the swept volume of the L.P cylinder

\[ v_{x1} = \frac{\pi}{4} (D_1)^2 L_1 = \frac{\pi}{4} (0.4)^2 0.5 \]

\[ = 0.06284 \text{ m}^3 \]

And volumetric efficiency,

\[ \eta_v = 1 + K - K (\frac{P_1}{P_2})^\frac{2}{3} \]

\[ = 1 + 0.04 - 0.04 (\frac{4.7}{1})^{\frac{1}{3}} \]

\[ = 0.9085 \text{ or } 90.85\% \]

Volume of air sucked by air pressure compressor,

\[ v_1 = v_{x1} \times \eta_v = 0.06284 \times 0.9085 = 0.0571 \frac{m^3}{\text{stroke}} \]

\[ = 0.0571 \times N_{strokes} = 0.0571 \times 200 = 1 \]

1.42m^3/min
And volume of air sucked by H.P compressor,

\[ v_3 = \frac{P_1 V_1}{P_3} = \frac{1 \times 11.42}{4.5} = 2.54 \text{ m}^3/\text{min} \]

We know that indicated work done by L.P compressor,

\[ W_L = \left( \frac{n}{n-1} \right) P_1 v_1 \left[ \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \]

\[ = \left( \frac{1.3}{1.3 - 1} \right) 1 \times 10^5 \times 11.42 \left[ \left( \frac{4.7}{4.13} \right)^{\frac{13-1}{13}} - 1 \right] \]

\[ = 2123.3 \times 10^3 \text{ J/min} = 2123.3 \text{ KJ/min} \]

And indicated work done by H.P compressor,

\[ W_H = \left( \frac{n}{n-1} \right) P_3 v_3 \left[ \left( \frac{P_4}{P_3} \right)^{\frac{n-1}{n}} - 1 \right] \]

\[ = \left( \frac{1.3}{1.3 - 1} \right) 4.5 \times 10^5 \times 2.54 \left[ \left( \frac{4.20}{4.5} \right)^{\frac{13-1}{13}} - 1 \right] \]

\[ = 2043.5 \times 10^3 \text{ J/min} = 2034.5 \text{ KJ/min} \]

Total indicated work done by the compressor,

\[ W = W_L + W_H = 2123.3 + 2034.5 = 4157.8 \text{ KJ/min} \]

Indicated power required to run the compressor

\[ = \frac{4157.8}{60} \]

\[ = 69.3 \text{ KW} \]
UNIT III INTERNAL COMBUSTION ENGINES AND COMBUSTION

- The most significance of IC engine in Day to day life, It is a lightweight and reasonably compact way to get power from fuel.
- It is also significantly safer and more efficient than the engine it replaced - steam.
- Small, lightweight IC engines made personal transportation possible. It transformed how we built cities and did business.

Classification of IC Engines:

Components and functions of IC Engines

The cylinder block is the main body of the engine, the structure that supports all the other components of the engine. In the case of the single cylinder engine the cylinder block houses the cylinder, while in the case of multi-cylinder engine the number of cylinders are cast together to form the cylinder block. The cylinder head is mounted at the top of the cylinder block. When the vehicle runs, large amounts of heat are generated within the cylinder block. To remove this heat the cylinder block and the cylinder head are cooled by water flowing through the water jackets within larger engines such as those found in cars and trucks. For smaller vehicles like motorcycles, fins are provided on the cylinder block and on the cylinder head to cool them. The bottom portion of the cylinder block is called a crankcase. Within the crankcase is where lubricating oil, which is used for lubricating various moving parts of the engine, is stored.

Cylinder:

As the name suggests it is a cylindrical shaped vessel fitted in the cylinder block. This
cylinder can be removed from the cylinder block and machined whenever required to. It is called the reciprocating motion of the piston. Burning of fuel occurs at the top of the cylinder, due to which the reciprocating motion of the piston is produced. The surface of the cylinder is finished to a high finish, so that there is minimal friction between the piston and the cylinder.

**Piston:**

The piston is the round cylindrical component that performs a reciprocating motion inside the cylinder. While the cylinder itself is the female part, the piston is the male part. The piston fits perfectly inside the cylinder. Piston rings are fitted over the piston. The gap between the piston and the cylinder is filled by the piston rings and lubricating oil. The piston is usually made up of aluminum

**Piston rings:**

The piston rings are thin rings fitted in the slots made along the surface of the piston. It provides a tight seal between the piston and the cylinder walls that prevents leaking of the combustion gases from one side to the other. This ensures that that motion of the piston produces as close as to the power generated from inside the cylinder.

**Combustion chamber:**

It is in the combustion chamber where the actual burning of fuel occurs. It is the uppermost portion of the cylinder enclosed by the cylinder head and the piston. When the fuel is burnt, much thermal energy is produced which generates excessively high pressures causing the reciprocating motion of the piston.

**Inlet manifold:**

Through the inlet manifold the air or air-fuel mixture is drawn into the cylinder.

**Exhaust manifold:**

All the exhaust gases generated inside the cylinder after burning of fuel are discharged through the exhaust manifold into the atmosphere.

**Inlet and exhaust valves:**

The inlet and the exhaust valves are placed at the top of the cylinder in the cylinder head. The inlet valve allows the intake of the fuel during suction stroke of the piston and to close thereafter. During the exhaust stroke of the piston the exhaust valves open allowing the exhaust gases to release to the atmosphere. Both these valves allow the flow of fuel and gases in single direction only.
**Spark plug:**

The spark plug is a device that produces a small spark that causes the instant burning of the pressurized fuel.

**Connecting rod:**

It is the connecting link between the piston and the crankshaft that performs the rotary motion. There are two ends of the connecting rod called the small end and big end. The small end of the connecting rod is connected to the piston by gudgeon pin, while the big end is connected to crankshaft by crank pin.

**Crankshaft:**

The crankshaft performs the rotary motion. It is connected to the axle of the wheels which move as the crankshaft rotates. The reciprocating motion of the piston is converted into the rotary motion of the crankshaft with the help of connecting rod. The crankshaft is located in the crankcase and it rotates in the bushings.

**Camshaft:**

It takes driving force from crankshaft through gear train or chain and operates the inlet valve as well as exhaust valve with the help of cam followers, push rod and rocker arms.

**Theoretical valve timing diagram of four stroke engine:**

![Theoretical Valve Timing Diagram](image-url)
2.7 Actual valve timing diagram of four stroke engine:

Fig 2.2 Actual valve timing diagram of four stroke engine

Theoretical port timing diagram of two stroke engine:

Fig 2.3 Theoretical port timing diagram of two stroke engine.
## Comparison of two stroke and four stroke engines

<table>
<thead>
<tr>
<th>Si No.</th>
<th>Four stroke Cycle engine</th>
<th>Two Stroke Cycle Engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>For every two revolutions of the crankshaft, there is one power stroke i.e., after every four piston strokes.</td>
<td>For every one revolution of the crankshaft, there is one power stroke i.e., after every two piston strokes.</td>
</tr>
<tr>
<td>2</td>
<td>For some power, more space is required.</td>
<td>For the same power less space is required.</td>
</tr>
<tr>
<td>3</td>
<td>Valves are required - inlet and exhaust valves.</td>
<td>Ports are made in the cylinder walls - inlet, exhaust and transfer port.</td>
</tr>
<tr>
<td>4</td>
<td>As the valves move frequently, lubrication is essential.</td>
<td>Arrangement of ports, reduce wear and tear and lubrication is not very essential.</td>
</tr>
<tr>
<td>5</td>
<td>Heavier flywheel is required because the turning moment (torque) of the crankshaft is not uniform i.e. one working stroke in every two revolution.</td>
<td>Lighter flywheel is required because the turning moment of the crankshaft is much more uniform i.e. one working stroke for every revolution.</td>
</tr>
<tr>
<td>6</td>
<td>These engines are water cooled, making it complicated in design and difficulty to maintain</td>
<td>These engines are generally air cooled, simple in design and easy to maintain.</td>
</tr>
<tr>
<td>7</td>
<td>The fuel-air change (mixture) is completely utilized thus efficiency is higher</td>
<td>As inlet and outlet port open simultaneous, some times fresh charge escapes with the exhaust gases are not always completely removed. This causes lower efficiency.</td>
</tr>
</tbody>
</table>
FUEL SYSTEMS

CARBURETOR:

FUNCTION:

- A carburetor is a device that blends air and fuel for an internal combustion engine.

PRINCIPLE:

- The carburetor works on Bernoulli's principle: the faster air moves, the lower its static pressure, and the higher its dynamic pressure.
- The throttle (accelerator) linkage does not directly control the flow of liquid fuel. Instead, it actuates carburetor mechanisms which meter the flow of air being pulled into the engine.
- The speed of this flow, and therefore its pressure, determines the amount of fuel drawn into the airstream.
- When carburetors are used in aircraft with piston engines, special designs and features are needed to prevent fuel starvation during inverted flight. Later engines used an early form of fuel injection known as a pressure carburetor.

FUEL INJECTION:

- Fuel injection is a system for admitting fuel into an internal combustion engine.
- It has become the primary fuel delivery system used in automotive engines.
- The primary difference between carburetors and fuel injection is that fuel injection atomizes the fuel by forcibly pumping it through a small nozzle under high pressure, while a carburetor relies on suction created by intake air accelerated through a Venturi tube to draw the fuel into the airstream.
- Fuel is transported from the fuel tank (via fuel lines) and pressurised using fuel pump(s). Maintaining the correct fuel pressure is done by a fuel pressure regulator.
2.10.3 FUEL INJECTOR

- When signalled by the engine control unit the fuel injector opens and sprays the pressurised fuel into the engine.
- The duration that the injector is open (called the pulse width) is proportional to the amount of fuel delivered.
- Depending on the system design, the timing of when injector opens is either relative each individual cylinder (for a sequential fuel injection system), or injectors for multiple cylinders may be signalled to open at the same time (in a batch fire system).

Direct injection

- In a direct injection engine, fuel is injected into the combustion chamber as opposed to injection before the intake valve (petrol engine) or a separate pre-combustion chamber (diesel engine).
- Direct fuel injection costs more than indirect injection systems: the injectors are exposed to more heat and pressure, so more costly materials and higher-precision electronic management systems are required.

Multiport fuel injection

- Multiport fuel injection injects fuel into the intake ports just upstream of each cylinder's intake valve, rather than at a central point within an intake manifold.
- The intake is only slightly wet, and typical fuel pressure runs between 40-60 psi.

Diesel knocking and detonation:

We already know that if the delay period is long, a large amount of fuel will be injected and accumulated in the chamber. The auto ignition of this large amount of fuel may cause high rate of pressure rise and high maximum pressure which may cause knocking in diesel engines. A long delay period not only increases the amount of fuel injected by the moment of ignition, but also improve the homogeneity of the fuel air mixture and its chemical preparedness for explosion type self ignition similar to detonation in SI engines. It is very instructive to compare the phenomenon of detonation is SI ensues with that of knocking in CI engines. There is no doubt that these two phenomena are fundamentally similar. Both are processes of auto ignition subject to the ignition time lag characteristic of the fuel air mixture. However, differences in the knocking phenomena of the SI engine and the CI engine should also be care fully be noted: 1. In the SI engine, the detonation occurs near the end of combustion where as in the CI engine detonation occurs near the beginning of combustion as shown in fig.
In the CI engine the fuel and air are in perfectly mixed and hence the rate of pressure rise is normally lower than that in the detonating part of the charge in the SI engine.

3. Since in the CI engine the fuel is injected into the cylinder only at the end of the compression stroke there is no question of pre ignition or pre mature ignition as in the SI engine.

4. In the SI engine it is relatively easy to distinguish between knocking and non-knocking operation as the human ear easily finds the distinction. However, in the case of the CI engine the normal ignition is itself by auto ignition and hence no CI engines have a sufficiently high rate of pressure rise per degree crank angle to cause audible noise.

When such noise becomes excessive or there is excessive vibration in engine structure, in the opinion of the observer, the engine is sending to knock. It is clear that personal judgment is involved here. Thus in the CI engine there is no definite distinction between normal and knocking combustion. The maximum rate of pressure rise in the CI engine may reach as high as 10 bar per crank degree angle.

It is most important to note that factors that tend to reduce detonation in the SI engine increase knocking in CI engine and vice versa because of the following reason. The detonation of knocking in the SI engine is due to simultaneous auto ignition of the last part of the charge. To eliminate detonation in the SI engine we want to prevent all together the auto ignition of the last part of the charge and therefore desire a long delay period and high self ignition temperature of the fuel. To eliminate knocking the CI engine we want to achieve auto ignitions early as possible therefore desire a short delay period and low self ignition temperature of the fuel. Table 6.2 gives the factors which reduce knocking in the SI and CI engines.
IGNITION SYSTEM

Basically Conventional Ignition systems are of 2 types: (a) Battery or Coil Ignition System, and (b) Magneto Ignition System. Both these conventional, ignition systems work on mutual electromagnetic induction principle. Battery ignition system was generally used in 4-wheelers, but now-a-days it is more commonly used in 2-wheelers also (i.e. Button start, 2-wheelers like Pulsar, Kinetic Honda; Honda-Activia, Scooty, Fiero, etc.). In this case 6 V or 12 V batteries will supply necessary current in the primary winding. Magneto ignition system is mainly used in 2-wheelers, kick this case magneto will produce and supply current to the primary winding. So in magneto ignition system magneto replaces the battery. Battery or Coil Ignition System

Figure shows line diagram of battery ignition system for a 4-cylinder petrol engine.

It mainly consists of a 6 or 12 volt battery, ammeter, ignition switch, auto-transformer (step up transformer), contact breaker, capacitor, distributor rotor, distributor contact points, spark plugs, etc. Note that the Figure 4.1 shows the ignition system for 4-cylinder petrol engine, here there are 4-spark plugs and contact breaker cam has 4 corners. (If it is for 6-cylinder engine it will have 6-spark plugs and contact breaker cam will be a hexagon).

The ignition system is divided into 2-circuits:

i. Primary Circuit:

a. It consists of 6 or 12 V battery, ammeter, ignition switch, primary winding it has 200-300 turns of 20 SWG (Sharps Wire Gauge) gauge wire, contact breaker, capacitor.
Fig 2.8 Ignition System
(ii) Secondary Circuit:

It consists of secondary winding. Secondary Ignition Systems winding consists of about 21000 turns of 40 (S WG) gauge wire. Bottom end of which is connected to bottom end of primary and top end of secondary winding is connected to centre of distributor rotor. Distributor rotors rotate and make contacts with contact points and are connected to spark plugs which are fitted in cylinder heads (engine earth).

(iii) Working: When the ignition switch is closed and engine in cranked, as soon as the contact breaker closes, a low voltage current will flow through the primary winding. It is also to be noted that the contact beaker cam opens and closes the circuit 4-times (for 4 cylinders) in one revolution. When the contact breaker opens the contact, the magnetic field begins to collapse. Because of this collapsing magnetic field, current will be induced in the secondary winding. And because of more turns (@ 21000 turns) of secondary, voltage goes unto 28000-30000 volts. This high voltage current is brought to centre of the distributor rotor. Distributor rotor rotates and supplies this high voltage current to proper spark plug depending upon the engine firing order. When the high voltage current jumps the spark plug gap, it produces the spark and the charge is ignited—combustion starts—products of combustion expand and produce power. Magneto Ignition System In this case magneto will produce and supply the required current to the primary winding. In this case as shown, we can have rotating magneto with fixed coil or rotating coil with fixed magneto for producing and supplying current to primary, remaining arrangement is same as that of a battery ignition system.
Comparison between Battery and Magneto Ignition System:

<table>
<thead>
<tr>
<th>Battery Ignition</th>
<th>Magneto Ignition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery is a must.</td>
<td>No battery needed.</td>
</tr>
<tr>
<td>Battery supplies current in primary circuit.</td>
<td>Magneto produces the required current for primary circuit.</td>
</tr>
<tr>
<td>A good spark is available at low speed also.</td>
<td>During starting the quality of spark is poor due to slow speed.</td>
</tr>
<tr>
<td>Occupies more space.</td>
<td>Very much compact.</td>
</tr>
<tr>
<td>Recharging is a must in case battery gets discharged.</td>
<td>No such arrangement required.</td>
</tr>
<tr>
<td>Mostly employed in car and bus for which it is required to crank the engine.</td>
<td>Used on motorcycles, scooters, etc.</td>
</tr>
<tr>
<td>Battery maintenance is required.</td>
<td>No battery maintenance problems.</td>
</tr>
</tbody>
</table>
What lubrication system does for an engine?

1. The job of the lubrication system is to distribute oil to the moving parts to reduce friction between surfaces which rub against each other.
2. An oil pump is located on the bottom of the engine.
3. The pump is driven by a worm gear off the main exhaust valve cam shaft.
4. The oil is pumped to the top of the engine inside a feed line.
5. Small holes in the feed line allow the oil to drip inside the crankcase.
6. The oil drips onto the pistons as they move in the cylinders, lubricating the surface between the piston and cylinder.
7. The oil then runs down inside the crankcase to the main bearings holding the crankshaft.
8. Oil is picked up and splashed onto the bearings to lubricate these surfaces.
9. Along the outside of the bottom of the crankcase is a collection tube which gathers up the used oil and returns it to the oil pump to be circulated again.

Purpose of Lubrication System

Lubricate

- Reduces Friction by creating a thin film (Clearance) between moving parts (Bearings and journals)

Seals

- The oil helps form a gastight seal between piston rings and cylinder walls (Reduces Blow-By)
- Internal oil leak (blow-by) will result in blue smoke at the tale pipe.

Cleans

- As it circulates through the engine, the oil picks up metal particles and carbon, and brings them back down to the pan.

Absorbs shock

- When heavy loads are imposed on the bearings, the oil helps to cushion the load.

Viscosity:

- Viscosity is a measure of oil’s resistance to flow.
- A low viscosity oil is thin and flows easily
- A high viscosity oil is thick and flows slowly.
- As oil heats up it becomes more viscous (Becomes thin)
Proper lubrication of an engine is a complex process.

Motor oil must perform many functions under many different operating conditions. The primary functions of oil are listed below:

1. Provide a barrier between moving parts to reduce friction, heat buildup, and wear.
2. Disperse heat. Friction from moving parts and combustion of fuel produce heat that must be carried away.
3. Absorb and suspend dirt and other particles. Dirt and carbon particles need to be carried by the oil to the oil filter where they can be trapped.
4. Neutralize acids that can build up and destroy polished metal surfaces.
5. Coat all engine parts. Oil should have the ability to leave a protective coating on all parts when the engine is turned off to prevent rust and corrosion.
6. Resist sludge and varnish buildup. Oil must be able to endure extreme heat without changing in physical properties or breaking down.
7. Stay fluid in cold weather; yet remain thick enough to offer engine

What the cooling system does for an engine.

1. Although gasoline engines have improved a lot, they are still not very efficient at turning chemical energy into mechanical power.
2. Most of the energy in the gasoline (perhaps 70%) is converted into heat, and it is the job of the cooling system to take care of that heat. In fact, the cooling system on a car driving down the freeway dissipates enough heat to heat two average-sized houses!
3. The primary job of the cooling system is to keep the engine from overheating by transferring this heat to the air, but the cooling system also has several other important jobs.
4. The engine in your car runs best at a fairly high temperature.
5. When the engine is cold, components wear out faster, and the engine is less efficient and emits more pollution.
6. So another important job of the cooling system is to allow the engine to heat up as quickly as possible, and then to keep the engine at a constant temperature.
2.14.1 Splash:

The splash system is no longer used in automotive engines. It is widely used in small four-cycle engines for lawn mowers, outboard marine operation, and so on. In the splash lubricating system, oil is splashed up from the oil pan or oil trays in the lower part of the crankcase. The oil is thrown upward as droplets or fine mist and provides adequate lubrication to valve mechanisms, piston pins, cylinder walls, and piston rings.

In the engine, dippers on the connecting-rod bearing caps enter the oil pan with each crankshaft revolution to produce the oil splash. A passage is drilled in each connecting rod from the dipper to the bearing to ensure lubrication. This system is too uncertain for automotive applications. One reason is that the level of oil in the crankcase will vary greatly the amount of lubrication received by the engine. A high level results in excess lubrication and oil consumption and a slightly low level results in inadequate lubrication and failure of the engine.

**Combination of Splash and Force Feed:**
In a combination splash and force feed, oil is delivered to some parts by means of splashing and other parts through oil passages under pressure from the oil pump. The oil from the pump enters the oil galleries.

From the oil galleries, it flows to the main bearings and camshaft bearings. The main bearings have oil-feed holes or grooves that feed oil into drilled passages in the crankshaft. The oil flows through these passages to the connecting rod bearings. From there, on some engines, it flows through holes drilled in the connecting rods to the piston-pin bearings. Cylinder walls are lubricated by splashing oil thrown off from the connecting-rod bearings. Some engines use small troughs under each connecting rod that are kept full by small nozzles which deliver oil under pressure from the oil pump. These oil nozzles deliver an increasingly heavy stream as speed increases.

At very high speeds these oil streams are powerful enough to strike the dippers directly. This causes a much heavier splash so that adequate lubrication of the pistons and the connecting-rod bearings is provided at higher speeds. If a combination system is used on an overhead valve engine, the upper valve train is lubricated by pressure from the pump.

**Force Feed:**

![Diagram](image.png)

A somewhat more complete pressurization of lubrication is achieved in the force-feed lubrication system. Oil is forced by the oil pump from the crankcase to the main
bearings and the camshaft bearings. Unlike the combination system the connecting-rod bearings are also fed oil under pressure from the pump. Oil passages are drilled in the crankshaft to lead oil to the connecting-rod bearings.

The passages deliver oil from the main bearing journals to the rod bearing journals. In some engines, these opening are holes that line up once for every crankshaft revolution. In other engines, there are annular grooves in the main bearings through which oil can feed constantly into the hole in the crankshaft. The pressurized oil that lubricates the connecting-rod bearings goes on to lubricate the pistons and walls by squirting out through strategically drilled holes. This lubrication system is used in virtually all engines that are equipped with semi floating piston pins.

**Full Force Feed:**

In a full force-feed lubrication system, the main bearings, rod bearings, camshaft bearings, and the complete valve mechanism are lubricated by oil under pressure. In addition, the full force-feed lubrication system provides lubrication under pressure to the pistons and the piston pins. This is accomplished by holes drilled the length of the connecting rod, creating an oil passage from the connecting rod bearing to the piston pin bearing. This passage not only feeds the piston pin bearings but also provides lubrication for the pistons and cylinder walls. This system is used in virtually all engines that are equipped with full-floating piston pins.

**Cooling System:**

**Air Cooled System:**

Air cooled system is generally used in small engines say up to 15-20 Kw and in airplane engines. In this system fins or extended surfaces are provided on the cylinder walls, cylinder head, etc. Heat generated due to combustion in the engine cylinder will be conducted to the fins and when the air flows over the fins, heat will be dissipated to air. The amount of heat dissipated to air depends upon: (a) Amount of air flowing through the fins. (b) Fin surface area. I Thermal conductivity of metal used for fins.
Advantages of Air Cooled System Following are the advantages of air cooled system:
(a) Radiator/pump is absent hence the system is light. (b) In case of water cooling system there are leakages, but in this case there are no leakages. I Coolant and antifreeze solutions are not required. (d) This system can be used in cold climates, where if water is used it may freeze.

Disadvantages of Air Cooled System (a) Comparatively it is less efficient. (b) It is used only in aero planes and motorcycle engines where the engines are exposed to air directly.

Water Cooling System:

In this method, cooling water jackets are provided around the cylinder, cylinder head, valve seats etc. The water when circulated through the jackets, it absorbs heat of combustion. This hot water will then be cooling in the radiator partially by a fan and partially by the flow developed by the forward motion of the vehicle. The cooled water is again recirculated through the water jackets

Thermo Syphon System:

In this system the circulation of water is due to difference in temperature (i.e. difference in densities) of water. So in this system pump is not required but water is circulated because of density difference only.
**Pump Circulation System:** In this system circulation of water is obtained by a pump. This pump is driven by means of engine output shaft through V-belts.

**Performance Calculation:**

Engine performance is an indication of the degree of success of the engine performs its assigned task, i.e. the conversion of the chemical energy contained in the fuel into the useful mechanical work. The performance of an engine is evaluated on the basis of the following: (a) Specific Fuel Consumption. (b) Brake Mean Effective Pressure. (c) Specific Power Output. (d) Specific Weight. (e) Exhaust Smoke and Other Emissions. The particular application of the engine decides the relative importance of these performance parameters. For Example: For an aircraft engine specific weight is more important whereas for an industrial engine specific fuel consumption is more important. For the evaluation of an engine performance few more parameters are chosen and the
effect of various operating conditions, design concepts and modifications on these parameters are studied. The basic performance parameters are the following:

(a) Power and Mechanical Efficiency.

(b) Mean Effective Pressure and Torque.

(c) Specific Output.

(d) Volumetric Efficiency.

(e) Fuel-air Ratio.

(f) Specific Fuel Consumption.

(g) Thermal Efficiency and Heat Balance.

(h) Exhaust Smoke and Other Emissions.

(i) Specific Weight.

**Power and Mechanical Efficiency** The main purpose of running an engine is to obtain mechanical power. • Power is defined as the rate of doing work and is equal to the product of force and linear velocity or the product of torque and angular velocity. • Thus, the measurement of power involves the measurement of force (or torque) as well as speed. The force or torque is measured with the help of a dynamometer and the speed by a tachometer. The power developed by an engine and measured at the output shaft is called the brake power (bp) and is given by bp=2Πnt/60 where, T is torque in N-m and N is the rotational speed in revolutions per minute.

The total power developed by combustion of fuel in the combustion chamber is, however, more than the bp and is called indicated power (ip). Of the power developed by the engine, i.e. ip, some power is consumed in overcoming the friction between moving parts, some in the process of inducting the air and removing the products of combustion from the engine combustion chamber.

**Indicated Power:** It is the power developed in the cylinder and thus, forms the basis of evaluation of combustion efficiency or the heat release in the cylinder. Where, I.P= PmLANK/60

\[
\text{pm} = \text{Mean effective pressure, N/m}^2, \ L = \text{Length of the stroke, m, A} = \text{Area of the piston, m}^2, \ N = \text{Rotational speed of the engine, rpm (It is N/2 for four stroke engine), and k = Number of cylinders.}
\]

Thus, we see that for a given engine the power output can be measured in terms of mean effective pressure. The difference between the ip and bp is the indication of the power lost in the mechanical components of the engine (due to friction) and forms the basis of mechanical efficiency; which is defined as follows: Mechanical efficiency=bp/ip The difference between ip and bp is called friction power (fp). Fp = ip − bp Mechanical efficiency= b.p/(bp+fp)
**Mean Effective Pressure and Torque:** Mean effective pressure is defined as a hypothetical/average pressure which is assumed to be acting on the piston throughout the power stroke. Therefore, \( P_m = \frac{60 \times I_p}{L \times A \times N \times k} \) where, \( P_m \) = Mean effective pressure, \( N/m^2 \), \( I_p \) = Indicated power, Watt, \( L \) = Length of the stroke, m, \( A \) = Area of the piston, \( m^2 \), \( N \) = Rotational speed of the engine, rpm (It is \( N/2 \) for four stroke engine), and \( k \) = Number of cylinders. If the mean effective pressure is based on bp it is called the brake mean effective pressure (Pm), and if based on ihp it is called indicated mean effective pressure (imep). Similarly, the friction mean effective pressure (fmep) can be defined as, \( f_{mep} = \text{imep} - \text{bme}_p \). The torque is related to mean effective pressure by the relation \( B.P = \frac{2\pi nt}{60} \) \( I.P = \frac{P_m L A N k}{60} \)

Thus, the torque and the mean effective pressure are related by the engine size. A large engine produces more torque for the same mean effective pressure. For this reason, torque is not the measure of the ability of an engine to utilize its displacement for producing power from fuel. It is the mean effective pressure which gives an indication of engine displacement utilization for this conversion. Higher the mean effective pressure, higher will be the power developed by the engine for a given displacement. Again we see that the power of an engine is dependent on its size and speed.

Therefore, it is not possible to compare engines on the basis of either power or torque. Mean effective pressure is the true indication of the relative performance of different engines.

**Specific Output:** Specific output of an engine is defined as the brake power (output) per unit of piston displacement and is given by, \( \text{Specific output} = \frac{B.P}{A.L \text{ Constant}} = \text{bme}_p \times \text{rpm} \). The specific output consists of two elements – the \( \text{bme}_p \) (force) available to work and the speed with which it is working. Therefore, for the same piston displacement and \( \text{bme}_p \) an engine operating at higher speed will give more output. It is clear that the output of an engine can be increased by increasing either speed or \( \text{bme}_p \). Increasing speed involves increase in the mechanical stress of various engine parts whereas increasing \( \text{bme}_p \) requires better heat release and more load on engine cylinder.
**Volumetric Efficiency:** Volumetric efficiency of an engine is an indication of the measure of the degree to which the engine fills its swept volume. It is defined as the ratio of the mass of air inducted into the engine cylinder during the suction stroke to the mass of the air corresponding to the swept volume of the engine at atmospheric pressure and temperature. Alternatively, it can be defined as the ratio of the actual volume inhaled during suction stroke measured at intake conditions to the swept volume of the piston. Volumetric efficiency, $h_v = \frac{\text{Mass of charge actually sucked in}}{\text{Mass of charge corresponding to the cylinder intake}}$. The amount of air taken inside the cylinder is dependent on the volumetric efficiency of an engine and hence puts a limit on the amount of fuel which can be efficiently burned and the power output. For supercharged engine the volumetric efficiency has no meaning as it comes out to be more than unity.

**Fuel-Air Ratio (F/A):** Fuel-air ratio (F/A) is the ratio of the mass of fuel to the mass of air in the fuel-air mixture. Air-fuel ratio (A/F) is reciprocal of fuel-air ratio. Fuel-air ratio of the mixture affects the combustion phenomenon in that it determines the flame propagation velocity, the heat release in the combustion chamber, the maximum temperature and the completeness of combustion. Relative fuel-air ratio is defined as the ratio of the actual fuel-air ratio to that of the stoichiometric fuel-air ratio required to burn the fuel supplied. Stoichiometric fuel-air ratio is the ratio of fuel to air is one in which case fuel is completely burned due to minimum quantity of air supplied. Relative fuel-air ratio, 

$\frac{\text{Actual Fuel Air ratio}}{\text{Stoichiometric fuel Air ratio}}$

**Brake Specific Fuel Consumption:** Specific fuel consumption is defined as the amount of fuel consumed for each unit of brake power developed per hour. It is a clear indication of the efficiency with which the engine develops power from fuel. B.S.F.C= Relative fuel-air ratio, 

$\frac{\text{Actual Fuel Air ratio}}{\text{Stoichiometric fuel Air ratio}}$. This parameter is widely used to compare the performance of different engines.

**Thermal Efficiency and Heat Balance:** Thermal efficiency of an engine is defined as the ratio of the output to that of the chemical energy input in the form of fuel supply. It may be based on brake or indicated output. It is the true indication of the efficiency with which the chemical energy of fuel (input) is converted into mechanical work.
Thermal efficiency also accounts for combustion efficiency, i.e., for the fact that whole of the chemical energy of the fuel is not converted into heat energy during combustion. Brake thermal efficiency = B.P/mf* Cv where, Cv = Calorific value of fuel, Kj/kg, and mf = Mass of fuel supplied, kg/sec. • The energy input to the engine goes out in various forms – a part is in the form of brake output, a part into exhaust, and the rest is taken by cooling water and the lubricating oil. • The break-up of the total energy input into these different parts is called the heat balance. • The main components in a heat balance are brake output, coolant losses, heat going to exhaust, radiation and other losses. • Preparation of heat balance sheet gives us an idea about the amount of energy wasted in various parts and allows us to think of methods to reduce the losses so incurred.

**Exhaust Smoke and Other Emissions:**

Smoke and other exhaust emissions such as oxides of nitrogen, unburned hydrocarbons, etc. are nuisance for the public environment. With increasing emphasis on air pollution control all efforts are being made to keep them as minimum as it could be. Smoke is an indication of incomplete combustion. It limits the output of an engine if air pollution control is the consideration.

**Emission Formation Mechanisms: (S.I)** This section discusses the formation of HC, CO, Nox, CO2, and aldehydes and explains the effects of design parameters.

**Hydrocarbon Emissions:**

HC emissions are various compounds of hydrogen, carbon, and sometimes oxygen. They are burned or partially burned fuel and/or oil. HC emissions contribute to photochemical smog, ozone, and eye irritation.

There are several formation mechanisms for HC, and it is convenient to think about ways HC can avoid combustion and ways HC can be removed; we will discuss each below. Of course, most of the HC input is fuel, and most of it is burned during “normal” combustion. However, some HC avoids oxidation during this process. The processes by which fuel compounds escape burning during normal S.I. combustion are:

1. Fuel vapor-air mixture is compressed into the combustion chamber crevice volumes.
2. Fuel compounds are absorbed into oil layers on the cylinder liner.
3. Fuel is absorbed by and/or contained within deposits on the piston head and piston crown.
4. Quench layers on the combustion chamber wall are left as the flame extinguishes close to the
walls. 5. Fuel vapor-air mixture can be left unburned if the flame extinguishes before reaching the walls. 6. Liquid fuel within the cylinder may not evaporate and mix with sufficient air to burn prior to the end of combustion. 7. The mixture may leak through the exhaust valve seat. (ii) **Carbon Monoxide** Formation of CO is well established. Under some conditions, there is not enough O2 available for complete oxidation and some of the carbon in the fuel ends up as CO. The amount of CO, for a range of fuel composition and C/H ratios, is a function of the relative air-fuel ratio. Even when enough oxygen is present, high peak temperatures can cause dissociation – chemical combustion reactions in which carbon dioxide and water vapor separate into CO, H2, and O2. Conversion of CO to CO2 is governed by reaction CO + OH ↔ CO2 + H. Dissociated CO may freeze during the expansion stroke. (iii) **Oxides of Nitrogen** Nox is a generic term for the compounds NO and NO2. Both are present to some degree in the exhaust, and NO oxidizes to NO2 in the atmosphere. Nox contributes to acid rain and photochemical smog; it is also thought to cause respiratory health problems at atmospheric concentrations found in some parts of the world. To understand Nox formation, we must recognize several factors that affect Nox equilibrium. Remember that all chemical reactions proceed toward equilibrium at some reaction rate. Equilibrium NO (which comprises most of the Nox formation) is formed at a rate that varies strongly with temperature and equivalence ratio.

(iv) **Carbon Dioxide** While not normally considered a pollutant, CO2 may contribute to the greenhouse effect. Proposals to reduce CO2 emissions have been made. CO2 controls strongly influence fuel economy requirements. (v) **Aldehydes** Aldehydes are the result of partial oxidation of alcohols. They are not usually present in significant quantities in gasoline-fueled engines, but they are an issue when alcohol fuels are used. Aldehydes are thought to cause lung problems. So far, little information of engine calibration effects on aldehyde formation is available.

**Emission Formation in C.I. Engine:**

For many years, diesel engines have had a reputation of giving poor performance and producing black smoke, an unpleasant odor, and considerable noise. However, it would find it difficult to distinguish today’s modern diesel car from its gasoline counterpart. For diesel engines the emphasis is to reduce emissions of Nox and particulates, where these emissions are typically higher than those from equivalent port injected gasoline engines equipped with three-way catalysts. Catalyst of diesel
exhaust remains a problem insofar as research has not yet been able to come up with an effective converter that eliminates both particulate matter (PM) and oxide of nitrogen.

**TECHNICAL TERMS**

**Alternator:** A generator producing alternating current used for recharging the vehicle battery.

**BMEP:** Pressure in I.C engine cylinder during the work stroke.

**Cam shaft:** A shaft having number of cams at appropriate angular position for opening the valves at timing relative to the piston movement.

**Carburation:** Air fuel mixing of correct strength

**Catalytic converter:** Convert toxic gases produced by I.C engines

**Clearance volume:** volume of engine cylinder above the piston when it is in the TDC position.

**Combustion chamber:** The small space in the engine cylinder head and or piston into which air fuel mixture (petrol engine) or air (diesel engine) is compressed and burnt.

**Connecting rod:** it converts the linear motion of the piston into the rotary motion of the crank shaft.

**EGR system:** Exhaust gas recirculation

**Fly wheel:** A heavy metallic wheel attached to the engine crankshaft to smoothen out the power surges from the engine power strokes.

**Governor:** A mechanical or electronic device to restrict the performance of an engine usually for reason of safety.

**Idle screw:** A screw on the carburetor for adjusting the idling speed of the engine.

**Detonation:** An uncontrolled explosion of the unburnt air fuel mixture in the engine cylinder.

**Pre-ignition:** Ignition of air-fuel mixture earlier than the spark plug.
Solved Problems:

1. A trial carried out in a four stroke single cylinder gas engine gave the following results. Cylinder dia=300mm, Engine stroke=500mm, Clearance volume=6750cc, Explosions per minute=100, \( P_{\text{min}} = 765 \text{ KN/m}^2 \) Net work load on the brake=190kg Brake dia=1.5m Rope dia=25mm, Speed of the engine=240rpm, Gas used=30 m³/kg hr, Calorific value of gas=20515 KJ/. Determine compression ratio, mechanical efficiency, indicated thermal efficiency, air standard efficiency, relative efficiency, assume \( \gamma = 1.4 \)

GIVEN DATA:-

- Dia of cylinder (d)=300mm=0.3m
- Engine stroke(l)=500mm=0.5m
- Clearance volume\( (v_c) = 6750/100^3 = 6.75 \times 10^{-3} \text{ m}^3 \)
- Explosions per minute\( (n) = 100/\text{minute} = 1.67/\text{sec} \)
- \( P_{\text{min}} = 765 \text{ KN/m}^2 \)
- Brake drum dia\( (D_1) = 1.5 \text{ m} \)
- Rope dia\( (d_1) = 0.025 \text{ m} \)
- Work load on the brake\( (w) = 190 \text{ kg} = 1.86 \text{ KN} \)

TO FIND:-

- Compression ratio \( (r) \)
- Mechanical efficiency \( (\eta_{\text{mech}}) \)
- Indicated thermal efficiency \( (\eta_i) \)
- Air standard efficiency \( (\eta_{\text{air}}) \)
- Relative efficiency \( (\eta_{\text{rel}}) \)
SOLUTION:-

(1). Compression Ratio (r):

\[ r = \left( \frac{V_1}{V_2} \right) + 1 \]

\[ = \left( \frac{4}{6.23} \right) + 1 \]

\[ = \frac{0.63}{0.75 \times 10^{-3}} + 1 \]

(2). Air Standard Efficiency (\( \eta_{\text{air}} \)):

\[ (r) = 6.23 \]

\[ \eta_{\text{air}} = 1 - \left( \frac{1}{r} \right) \]

\[ = 1 - \left( \frac{1}{6.23 - 1} \right) \]

= 51.89%

(3). Indicated Thermal Efficiency (\( \eta_{\text{it}} \)):

\[ (\eta_{\text{it}}) = \frac{\text{IP}}{F_c \times C_y} \]

Here, indicated power

\[ (\text{IP}) = p_{mi} \times l \times a \times n \times k \]

\[ = 765 \times 0.5 \times 0.0706 \times 1.67 \times 1 \]

\[ = 45.09 \text{KW} \]

Therefore,

\[ \eta_{\text{it}} = \frac{\text{IP}}{(\frac{\text{SU}}{22515}) \times 22515} \]

= 24.03%
(4). Relative Efficiency ($\eta_{rel}$):

\[ (\eta_{rel}) = \frac{\eta_{it}}{\eta_{air}} \]

\[ = \frac{41.8}{51.8} \]

\[ = 46.30\% \]

(5). Mechanical Efficiency ($\eta_{mech}$):

\[ (\eta_{mech}) = \frac{\text{B.H.P}}{\text{fuel}} \]

\[ = \frac{20000}{2400} \]

\[ = 79.02\% \]

2. The following observations are recorded during a test on a four-stroke petrol engine, F.C = 3000 of fuel in 12 sec, speed of the engine is 2500 rpm, B.P = 20KW, Air intake orifice diameter = 35 mm, Pressure across the orifice = 140 mm of water, coefficient of discharge of orifice = 0.6, piston diameter = 150 mm, stroke length = 100 mm, Density of the fuel = 0.85 gm/cc, r = 6.5, C_v of fuel = 42000 KJ/Kg, Barometric pressure = 760 mm of Hg, Room temperature = 24°C

Determine:

(i) Volumetric efficiency on the air basis alone

(ii) Air-fuel ratio

(iii) The brake mean effective pressure

(iv) The relative efficiency on the brake thermal efficiency

Given data:

Fuel consumption = 30 cc in 12 sec = \[ \frac{30}{12} \times 3600 \text{(cc/min)} \]

Speed (N) = 2500/60 rps Brake
power = 20KW
Orifice diameter ($d_0$) = 0.035 m
Pressure across the orifice ($P_o$) = 140 mm of water Coefficient of discharge ($C_d$) = 0.6
Piston diameter \((d) = 150\text{mm} = 0.15\text{m}\)

Stroke length \((l) = 0.1\text{m}\)

Density of fuel \((\rho) = 0.85\text{gm/cc}\)

Compression ratio \((r) = 6.5\)

Room temperature \((T_a) = 297\text{K}\)

Barometric pressure = 760mm of Hg = 101.325KPa/m^2 = 10.34m of water

To Find:

(i) Volumetric efficiency on the air basis alone 
(ii) Air-fuel ratio
(iii) The brake mean effective pressure
(iv) The relative efficiency on the brake thermal efficiency

Solution:

\[
10.34\text{m of water} = 101.325\text{KN/m}^2
\]

Pressure head \[
\frac{P_o}{\rho \times g} = 0.14\text{m of water}
\]

\[
P_o = 0.14 \times 0.14 = 0.0196
\]

\[
P_o = 1372\text{N/m}^2
\]

Density of gas \((\rho) = \frac{P}{RT}\)

\[
\rho = \frac{101.325}{0.287 \times 297} = 1.1887\text{Kg/m}^3
\]

Pressure head \((h) = \frac{1372}{1.1887 \times 9.81}
\]

\[
h = 117.6557\text{m}
\]

\[
Q_{air} = c_d \times a \times \sqrt{2gh}
\]
No. of Suction strokes per second = \( \frac{N}{2} = \frac{120}{60 \times 2} = 20.83 \text{ sec} \)

Air consumptions per stroke = \( \frac{0.02774}{20.83} \) = 0.001332 m³

Stroke volume (Vₜ) = \( \frac{\pi}{4} \times (0.15)^2 \times 0.1 = 0.001767 \text{ m}^3 \)

Volumetric efficiency (\( \eta_{vol} \)) = \( \frac{0.001332}{0.001767} \times 100\% \)

\( \eta_{vol} = 75.382\% \)

Volume of air consumed \( V_{air} = Q_{air} = 0.02774 \text{ m}^3/\text{sec} \)

\( = 0.02774 \times 3600 \text{ m}^3/\text{hr} \)

Mass of air consumed (\( m_a \)) = \( V_a \times 299.864 \times 1.1887 \)

\( = 118.71 \text{ Kg/hr} \)

Fuel consumption = 9000cc/hr

Mass of the fuel consumed (\( m_f \)) = 9000 \times 0.85 = 7.65 Kg/hr Air fuel ratio

\( \frac{m_a}{m_f} = \frac{118.71}{7.65} = 15.518 : 1 \)

Brake power (B.P) = 20KW = Pₘₖ \times 1 \times a \times n \times k

\( P_{mb} = \frac{20}{0.001 \times 67 \times 20.83 \times 1} \)

\( = 543.294 \text{ KN/m}^2 \)

Air standard efficiency (\( \eta_{air} \)) = \( 1 - \frac{1}{(r)^{y-1}} \)

\( = 1 - \frac{1}{(6.5)^{4.4-1}} \)

\( = 52.703\% \)

Brake thermal efficiency

\( (\eta_{BT}) = \frac{\text{B.P} \times M_r \times C_v}{\text{M}_f} \)

\( = 22.4\% \)
\[ \frac{F.C \times C.V}{\sqrt{2 \times 9.81 \times 117.6557}} \]

\[ = 0.02774 \text{ m}^3/\text{sec} \]

Relative efficiency on brake thermal efficiency basis \((\eta_{\text{rel}}) = \frac{\eta_{\text{BT}}}{\eta_{\text{air}}}\)
UNIT-5 GAS TURBINES

An Ideal Simple-Cycle Gas Turbine

The fundamental thermodynamic cycle on which gas turbine engines are based is called the *Brayton Cycle* or *Joule cycle*. A temperature-entropy diagram for this ideal cycle and its implementation as a *closed-cycle gas turbine* is shown in Figure 5.1. The cycle consists of an isentropic compression of the gas from state 1 to state 2; a constant pressure heat addition to state 3; an isentropic expansion to state 4, in which work is done; and an isobaric closure of the cycle back to state 1.

As Figure 5.1(a) shows, a compressor is connected to a turbine by a rotating shaft. The shaft transmits the power necessary to drive the compressor and delivers the balance to a power-utilizing load, such as an electrical generator. The turbine is similar in concept and in many features to the steam turbines discussed earlier, except that it is designed to extract power from a flowing hot gas rather than from water vapor. It is important to recognize at the outset that the term “gas turbine” has a dual usage: It designates both the entire engine and the device that drives the compressor and the load. It should be clear from the context which meaning is intended. The equivalent term “combustion turbine” is also used occasionally, with the same ambiguity.
Like the simple Rankine-cycle power plant, the gas turbine may be thought of as a device that operates between two pressure levels, as shown in Figure 5.1(b). The compressor raises the pressure and temperature of the incoming gas to the levels of \( p_2 \) and \( T_2 \). Expansion of the gas through the turbine back to the lower pressure at this point would be useless, because all the work produced in the expansion would be required to drive the compressor.
Instead, it is necessary to add heat and thus raise the temperature of the gas before expanding it in the turbine. This is achieved in the heater by heat transfer from an external source that raises the gas temperature to $T_3$, the turbine inlet temperature. Expansion of the hot gas through the turbine then delivers work in excess of that needed to drive the compressor. The turbine work exceeds the compressor requirement because the enthalpy differences, and hence the temperature differences, along isentropes connecting lines of constant pressure increase with increasing entropy (and temperature), as the figure suggests.

The difference between the turbine work, $W_t$, and the magnitude of the compressor work, $|W_c|$, is the net work of the cycle. The net work delivered at the output shaft may be used to drive an electric generator, to power a process compressor, turn an airplane propeller, or to provide mechanical power for some other useful activity.

In the closed-cycle gas turbine, the heater is a furnace in which combustion gases or a nuclear source transfer heat to the working fluid through thermally conducting tubes. It is sometimes useful to distinguish between internal and external combustion engines by whether combustion occurs in the working fluid or in an area separate from the working fluid, but in thermal contact with it. The combustion-heated, closed-cycle gas turbine is an example, like the steam power plant, of an external combustion engine. This is in contrast to internal combustion engines, such as automotive engines, in which combustion takes place in the working fluid confined between a cylinder and a piston, and in open-cycle gas turbines.

**Analysis of the Ideal Cycle**

The Air Standard cycle analysis is used here to review analytical techniques and to provide quantitative insights into the performance of an ideal-cycle engine. Air Standard cycle analysis treats the working fluid as a calorically perfect gas, that is, a perfect gas with constant specific heats evaluated at room temperature. In Air Standard cycle analysis the heat capacities used are those for air.

A gas turbine cycle is usually defined in terms of the compressor inlet pressure and temperature, $p_1$ and $T_1$, the compressor pressure ratio, $r = p_2/p_1$, and the turbine inlet temperature, $T_3$, where the subscripts correspond to states identified in Figure 5.1.

Starting with the compressor, its exit pressure is determined as the product of $p_1$ and the compressor pressure ratio. The compressor exit temperature may then be determined by the familiar relation for an isentropic process in an ideal gas, Equation (1.19):

$$T_2 = T_1 (p_2/p_1)^{(k-1)/k} \quad [R \mid K]$$

(5.1)

For the two isobaric processes, $p_2 = p_3$ and $p_4 = p_1$. Thus the turbine pressure ratio, $p_3/p_4$, is equal to the compressor pressure ratio, $r = p_2/p_1$. With the turbine inlet temperature $T_3$ known, the turbine discharge temperature can be determined from

$$T_4 = T_3 (p_2/p_1)^{(k-1)/k} \quad [R \mid K]$$

(5.2)
and the temperatures and pressures are then known at all the significant states.

Next, taking a control volume around the compressor, we determine the shaft work required by the compressor, \( w_c \), assuming negligible heat losses, by applying the steady-flow energy equation:

\[
0 = h_2 - h_1 + w_c
\]

or

\[
w_c = h_1 - h_2 = c_p (T_1 - T_2) \quad \text{[Btu/lbm | kJ/kg]} \quad (5.3)
\]

Similarly, for the turbine, the turbine work produced is

\[
w_t = h_3 - h_4 = c_p (T_3 - T_4) \quad \text{[Btu/lbm | kJ/kg]} \quad (5.4)
\]

The net work is then

\[
w_n = w_t + w_c = c_p (T_3 - T_4 + T_1 - T_2) \quad \text{[Btu/lbm | kJ/kg]} \quad (5.5)
\]

Now taking the control volume about the heater, we find that the heat addition per unit mass is

\[
q_a = h_3 - h_2 = c_p (T_3 - T_2) \quad \text{[Btu/lbm | kJ/kg]} \quad (5.6)
\]

The cycle thermal efficiency is the ratio of the net work to the heat supplied to the heater:

\[
y_{th} = \frac{w_n}{q_a} \quad \text{[dl]} \quad (5.7)
\]

which by substitution of Equations (5.1), (5.2), (5.5), and (5.6) may be simplified to

\[
y_{th} = 1 - \frac{p_2}{p_1} - \frac{(k-1)}{k} \quad \text{[dl]} \quad (5.8)
\]

It is evident from Equation (5.8) that increasing the compressor pressure ratio increases thermal efficiency.

Another parameter of great importance to the gas turbine is the work ratio, \( w_t / |w_c| \). This parameter should be as large as possible, because a large amount of the power delivered by the turbine is required to drive the compressor, and because the engine net work depends on the excess of the turbine work over the compressor work. A little algebra will show that the work ratio \( w_t / |w_c| \) can be written as:

\[
w_t / |w_c| = \frac{T_3}{T_1} / \left( \frac{p_2}{p_1} \right)^{(k-1)/k} \quad \text{[dl]} \quad (5.9)
\]
Note that, for the ideal cycle, the thermal efficiency and the work ratio depend on only two independent parameters, the compressor pressure ratio and the ratio of the turbine and compressor inlet temperatures. It will be seen that these two design parameters are of utmost importance for all gas turbine engines.

Equation (5.9) shows that the work ratio increases in direct proportion to the ratio \( T_3 / T_1 \) and inversely with a power of the pressure ratio. On the other hand, Equation (5.8) shows that thermal efficiency increases with increased pressure ratio. Thus, the desirability of high turbine inlet temperature and the necessity of a tradeoff involving pressure ratio is clear. Equation (5.9) also suggests that increases in the ratio \( T_3 / T_1 \) allow the compressor pressure ratio to be increased without reducing the work ratio.

This is indicative of the historic trend by which advances in materials allow higher turbine inlet temperatures and therefore higher compressor pressure ratios.

It was shown in Chapter 1 that the area of a reversible cycle plotted on a T–s diagram gives the net work of the cycle. With this in mind, it is interesting to consider a family of cycles in which the compressor inlet state, \( a \), and turbine inlet temperatures are fixed, as shown in Figure 5.2. As the compressor pressure ratio \( p_b/p_a \) approaches 1, the cycle area and hence the net work approach 0, as suggested by the shaded cycle labeled with single primes. At the other extreme, as the compressor pressure ratio approaches its maximum value, the net work also approaches 0, as in the cycle denoted by double primes. For intermediate pressure ratios, the net work is large and positive, indicating that there is a unique value of compressor pressure ratio that maximizes the net work. Such information is of great significance in gas turbine design,
because it indicates the pressure ratio that yields the highest power output for given turbomachine inlet temperatures and mass flow rate. This is an important approach to the pressure ratio tradeoff mentioned earlier. It will be considered from an analytic viewpoint for a more realistic gas turbine model in a later section.

Up to this point the discussion has focused on the closed-cycle gas turbine, an external combustion or nuclear-heated machine that operates with a circulating fixed mass of working fluid in a true cyclic process. In fact, the same Air Standard cycle analysis may be applied to the open-cycle gas turbine. The open cycle operates with atmospheric air that is pressurized by the compressor and then flows into a combustion chamber, where it oxidizes a hydrocarbon fuel to produce a hot gas that drives the turbine. The turbine delivers work as in the closed cycle, but the exiting combustion gases pass into the atmosphere, as they must in all combustion processes.

A diagram of the cycle implementation is shown in Figure 5.3. Clearly, the open-cycle gas turbine is an internal combustion engine, like the automotive engine. Note that the diagram is consistent with Figure 5.1 and all the preceding equations in this chapter. This is true because (1) the atmosphere serves as an almost infinite source and sink that may be thought of as closing the cycle, and (2) the energy released by combustion has the same effect as the addition of external heat in raising the temperature of the gas to the turbine inlet temperature. A cutaway of an open-cycle utility gas turbine is presented in Figure 5.4.
Realistic Simple-Cycle Gas Turbine Analysis

The preceding analysis of the Air Standard cycle assumes perfect turbomachinery, an unachievable but meaningful ideal, and room-temperature heat capacities. Realistic quantitative performance information can be obtained by taking into account efficiencies of the compressor and the turbine, significant pressure losses, and more realistic thermal properties.

Properties for Gas Turbine Analysis

It is pointed out in reference 1 that accurate gas turbine analyses may be performed using constant heat capacities for both air and combustion gases. This appears to be a specialization of a method devised by Whittle (ref. 4). The following properties are therefore adopted for all gas turbine analyses in this book:

**Air:**

\[ c_p = 0.24 \text{ Btu/lbm-R} \quad \text{or} \quad 1.004 \text{ kJ/kg-K} \]

\[ k = 1.4 \quad \text{implies} \quad k/(k-1) = 3.5 \]

**Combustion gas:**

\[ c_{p,g} = 0.2744 \text{ Btu/lbm-R} \quad \text{or} \quad 1.148 \text{ kJ/kg-K} \]

\[ k_g = 1.333 \quad \text{implies} \quad k_g/(k_g-1) = 4.0 \]

The properties labeled as combustion gas above are actually high-temperature-air properties. Because of the high air-fuel ratio required by gas turbines, the
thermodynamic properties of gas turbine combustion gases usually differ little from those of high-temperature air. Thus the results given below apply equally well to closed-cycle machines using air as the working fluid and to open-cycle engines.

**Analysis of the Open Simple-Cycle Gas Turbine**

A simple-cycle gas turbine has one turbine driving one compressor and a power-consuming load. More complex configurations are discussed later. It is assumed that the compressor inlet state, the compressor pressure ratio, and the turbine inlet temperature are known, as before. The turbine inlet temperature is usually determined by the limitations of the high-temperature turbine blade material. Special metals or ceramics are usually selected for their ability to withstand both high stress at elevated temperature and erosion and corrosion caused by undesirable components of the fuel.

As shown in Figure 5.3, air enters the compressor at a state defined by \( T_1 \) and \( p_1 \).

The compressor exit pressure, \( p_2 \), is given by

\[
p_2 = rp_1 \quad \text{[lb/ft}^2 \text{ | kPa]} \tag{5.10}
\]

where \( r \) is the compressor pressure ratio. The ideal compressor discharge temperature, \( T_{2s} \), is given by the isentropic relation

\[
T_{2s} = T_1 r^{(k-1)/k} \quad \text{[R | K]} \tag{5.11}
\]

The compressor isentropic efficiency, defined as the ratio of the compressor isentropic work to the actual compressor work with both starting at the same initial state and ending at the same pressure level, may be written as

\[
y_c = \frac{(h_1 - h_{2s})/(h_1 - h_2)}{(T_1 - T_{2s})/(T_1 - T_2)} \quad \text{[dl]} \tag{5.12}
\]

Here the steady-flow energy equation has been applied to obtain expressions for the work for an irreversible adiabatic compressor in the denominator and for an isentropic compressor in the numerator. Solving Equation (5.12) for \( T_2 \), we get as the actual compressor discharge temperature:

\[
T_2 = T_1 + \frac{(T_{2s} - T_1)}{y_c} \quad \text{[R | K]} \tag{5.13}
\]

Equation (5.3) then gives the work needed by the compressor, \( w_c \):

\[
w_c = c_p (T_1 - T_2) = c_p (T_1 - T_{2s}) / y_c \quad \text{[Btu/lbm | kJ/kg]} \tag{5.14}
\]

Note that the compressor work is negative, as required by the sign convention that defines work as positive if it is produced by the control volume. The compressor power requirement is, of course, then given by \( m_a w_c \text{ [Btu/hr | kW]} \), where \( m_a \) is the
compressor mass flow rate \([\text{lb}_m / \text{hr} \mid \text{kg} / \text{s}]\).

After leaving the compressor at an elevated pressure and temperature, the air then enters the combustion chamber, where it completely oxidizes a liquid or a gaseous fuel injected under pressure. The combustion process raises the combustion gas temperature to the turbine inlet temperature \(T_3\). One of the goals of combustion chamber design is to minimize the pressure loss from the compressor to the turbine. Ideally, then, \(p_3 = p_2\), as assumed by the Air Standard analysis. More realistically, a fixed value of the combustor fractional pressure loss, \(f_{pl}\), (perhaps about 0.05 or 5%) may be used to account for burner losses:

\[
f_{pl} = \frac{(p_2 - p_3)}{p_2} \quad [\text{dl}]
\]

Then the turbine inlet pressure may be determined from

\[
p_3 = (1 - f_{pl}) p_2 \quad [\text{lb}_f / \text{ft}^2 \mid \text{kPa}]
\]

Rather than deal with its complexities, we may view the combustion process simply as one in which heat released by exothermic chemical reaction raises the temperature of combustion gas (with hot-air properties) to the turbine inlet temperature. The rate of heat released by the combustion process may then be expressed as:

\[
Q_a = m_a (1 + f) c_{p,g} (T_3 - T_2) \quad [\text{Btu} / \text{hr} \mid \text{kW}]
\]

where \(f\) is the mass fuel-air ratio. The term \(m_a(1 + f)\) is seen to be the sum of the air and fuel mass flow rates, which also equals the mass flow rate of combustion gas. For gas turbines it will be seen later that \(f\) is usually much less than the stoichiometric fuel-air ratio and is often neglected with respect to 1 in preliminary analyses.

The turbine in the open-cycle engine operates between the pressure \(p_3\) and atmospheric pressure, \(p_4 = p_1\), with an inlet temperature of \(T_3\). If the turbine were isentropic, the discharge temperature would be

\[
T_{4s} = T_3 \left( \frac{p_4}{p_3} \right)^{\left(k_g - 1\right) / k_g} \quad [\text{R} \mid \text{K}]
\]

From the steady-flow energy equation, the turbine work can be written as

\[
w_t = c_{p,g} \left( T_3 - T_4 \right) = y_t c_{p,g} (T_3 - T_{4s}) [\text{Btu} / \text{lb}_m \mid \text{kJ/kg}]
\]

referred to unit mass of combustion gas, and where \(y_t\) is the turbine isentropic efficiency. The turbine power output is then \(m_a(1 + f)w_t\), where, as seen earlier, \(m_a(1 + f)\) is the mass flow rate of combustion gas flowing through the turbine. The net work based on the mass of air processed and the net power output of the gas turbine, \(P_n\), are then given by
\[ w_n = (1 + f)w_t + w_c \quad [\text{Btu/lbm air|kJ/kg air}] \quad (5.20) \]

and

\[ P_n = m_a [(1 + f)w_t + w_c] \quad [\text{Btu/hr | kW}] \quad (5.21) \]

and the thermal efficiency of the engine is

\[ \eta_{th} = \frac{P_n}{Q_a} \quad [\text{dl}] \quad (5.22) \]

**EXAMPLE 5.1**

A simple-cycle gas turbine has 86% and 89% compressor and turbine efficiencies, respectively, a compressor pressure ratio of 6, a 4% fractional pressure drop in the combustor, and a turbine inlet temperature of 1400°F. Ambient conditions are 60°F and one atmosphere. Determine the net work, thermal efficiency, and work ratio for the engine. Assume that the fuel-mass flow rate is negligible compared with the air flow rate.

**Solution**

The notation for the solution is that of Figure 5.3. The solution details are given in Table 5.1 in a step-by-step spreadsheet format. Each line presents the parameter name, symbol, and units of measure; its value; and the right-hand side of its specific determining equation.

**TABLE 5.1** Spreadsheet Solution to Example 5.1

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Symbol</th>
<th>Units of Measure</th>
<th>Value</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air isentropic exponent</td>
<td>k</td>
<td></td>
<td>1.40</td>
<td>given</td>
</tr>
<tr>
<td>(k-1)/k</td>
<td>e,g</td>
<td></td>
<td>0.2857</td>
<td>((k-1)/k)</td>
</tr>
<tr>
<td>Compressor inlet temperature</td>
<td>T1,R</td>
<td></td>
<td>520.00</td>
<td>given</td>
</tr>
<tr>
<td>Compressor efficiency</td>
<td>e_tac</td>
<td></td>
<td>0.86</td>
<td>given</td>
</tr>
<tr>
<td>Compressor pressure ratio</td>
<td>(r)</td>
<td></td>
<td>6.00</td>
<td>given</td>
</tr>
<tr>
<td>Compressor isentropic exit temp.</td>
<td>T2a,R</td>
<td></td>
<td>861.63</td>
<td>(T_1 r^e,g)</td>
</tr>
<tr>
<td>Compressor true exit temp.</td>
<td>T2,R</td>
<td></td>
<td>924.22</td>
<td>(T_1 + (T_2a - T_1) / e_{tac})</td>
</tr>
<tr>
<td>Compressor work</td>
<td>Wc, Btu/lb</td>
<td></td>
<td>-97.01</td>
<td>0.24(T1 - T2)</td>
</tr>
<tr>
<td>Combustor fractional pressure drop</td>
<td>f_p1</td>
<td></td>
<td>0.04</td>
<td>given</td>
</tr>
<tr>
<td>Turbine pressure ratio</td>
<td>(r_t)</td>
<td></td>
<td>5.76</td>
<td>(r(1-f_p1))</td>
</tr>
<tr>
<td>Turbine inlet temp.</td>
<td>T3,R</td>
<td></td>
<td>1860.00</td>
<td>given</td>
</tr>
<tr>
<td>Hot gas isentropic exponent</td>
<td>kg</td>
<td></td>
<td>1.33</td>
<td>given</td>
</tr>
<tr>
<td>(kg-1)/kg</td>
<td>e,g</td>
<td></td>
<td>0.25</td>
<td>((kg-1)/kg)</td>
</tr>
<tr>
<td>Hot gas heat capacity</td>
<td>cp,g, Btu/lb-R</td>
<td></td>
<td>0.2744</td>
<td>given</td>
</tr>
<tr>
<td>Combustor heat addition</td>
<td>Qa, Btu/lb</td>
<td></td>
<td>256.78</td>
<td>(cp_g(T_3-T_2))</td>
</tr>
<tr>
<td>Turbine isentropic exit temp.</td>
<td>T4s,R</td>
<td></td>
<td>1200.63</td>
<td>(T_3/(rt)^e,g)</td>
</tr>
<tr>
<td>Turbine isentropic efficiency</td>
<td>etac</td>
<td></td>
<td>0.89</td>
<td>given</td>
</tr>
<tr>
<td>Turbine true exit temp.</td>
<td>T4,R</td>
<td></td>
<td>1273.16</td>
<td>(T_3 - e_{tac}(T_3-T_4))</td>
</tr>
<tr>
<td>Turbine work</td>
<td>Wt, Btu/lb</td>
<td></td>
<td>161.03</td>
<td>(cp_g(T_3-T_4))</td>
</tr>
<tr>
<td>Net work</td>
<td>Wn, Btu/lb</td>
<td></td>
<td>64.02</td>
<td>(W_t+W_c)</td>
</tr>
<tr>
<td>Thermal efficiency</td>
<td>(\eta_{th})</td>
<td></td>
<td>0.25</td>
<td>(W_n/Q_a)</td>
</tr>
<tr>
<td>Work ratio</td>
<td></td>
<td></td>
<td>1.66</td>
<td>(W_n/W_c)</td>
</tr>
</tbody>
</table>
When an entire cycle is to be analyzed, it is best to start at the compressor with the inlet conditions and proceed to calculate successive data in the clockwise direction on the T-s diagram. The compressor isentropic and actual discharge temperatures and work are determined first using Equations (5.11), (5.13), and (5.14). The turbine pressure ratio is determined next, accounting for the combustor pressure loss, using Equation (5.16). The isentropic relation, Equation (5.18), gives the isentropic turbine exit temperature, and the turbine efficiency and Equation (5.19) yields the true turbine exit temperature and work. Once all the turbomachine inlet and exit temperatures are known, other cycle parameters are easily determined, such as the combustor heat transfer, net work, thermal efficiency, and work ratio.

An important observation may be made on the basis of this analysis regarding the magnitude of the compressor work with respect to the turbine work. Much of the turbine work is required to drive the compressor. Compare the work ratio of 1.66, for example, with the much higher values for the steam cycles of Chapter 2 (the Rankine- cycle pumps have the same function there as the compressor here). Example 2.4 for the Rankine cycle with a 90% turbine efficiency has a work ratio of 77.2. Thus the gas turbine’s pressurization handicap relative to the Rankine cycle is substantial.

The unimpressive value of the thermal efficiency of the example gas turbine, 25% (not typical of the current state of the art) compares with a Carnot efficiency for the same cycle temperature extremes of 72% The large amount of compressor work required clearly contributes to this weak performance. Nevertheless, current gas turbines are competitive with many other engines on an efficiency basis, and have advantages such as compactness and quick-start capability relative to Rankine cycle power plants. One approach to the improvement of thermal efficiency of the gas turbine will be addressed later in Section 5.5. First let’s look at what can be done about gas turbine work.

Maximizing the Net Work of the Cycle

Using Equations (5.14) and (5.19), we can rewrite the cycle net work as

\[ w_n = \frac{y_t c_p g (T_3 - T_{4s}) - c_p (T_{2s} - T_1)}{y_c} \quad \text{[Btu/lbm | kJ/kg]} \quad (5.23) \]

where the fuel-air ratio has been neglected with respect to 1. In the following, the combustor pressure losses and the distinction between hot-gas and air heat capacities will be neglected but the very important turbomachine efficiencies are retained.

Nondimensionalizing the net work with the constant \( c_p T_1 \) we get:

\[ \frac{w_n}{c_p T_1} = \frac{y_t (T_3 / T_1)(c_p g/c_p)(1 - r^{-(k-1)/k}) + (1 - r^{(k-1)/k})}{y_c} \quad \text{[dl]} \quad (5.24) \]

By differentiating \( w_n \) with respect to the compressor pressure ratio \( r \) and setting the
result equal to 0, we obtain an equation for \( r^* \), the value of \( r \) that maximizes the net work with fixed turbomachine efficiencies and with a constant ratio of the temperatures of the turbomachine inlets, \( T_3 / T_1 \). For constant gas properties throughout, the result is

\[
    r^* = \left( y_{c_l} y_T \frac{T_3}{T_1} \right)^{k/2(k-1)}
\]

This relation gives a specific value for the compressor pressure ratio that defines an optimum cycle, in the sense of the discussion of Figure 5.2. There it was established qualitatively that a cycle with maximum net work exists for a given value of \( T_3 / T_1 \). Equation (5.25) defines the condition for this maximum and generalizes it to include turbomachine inefficiency.

The pressure ratio \( r^* \) given by Equation (5.25) increases with increasing turbomachine efficiencies and with \( T_3 / T_1 \). This is a clear indicator that increasing turbine inlet temperature favors designs with higher compressor pressure ratios. This information is important to the gas turbine designer but does not tell the whole design story. There are other important considerations; for example, (1) compressors and turbines become more expensive with increasing pressure ratios, and (2) the pressure ratio that maximizes thermal efficiency is different from that given by Equation (5.25). Figure 5.5 shows the influence of compressor pressure ratio on both efficiency and net work and the position of the value given by Equation (5.25). Thus, when all factors are taken into account, the final design pressure ratio is likely to be in the vicinity of, but not necessarily identical to, \( r^* \).
Regenerative Gas Turbines

It was shown in Chapter 2 that the efficiency of the Rankine cycle could be improved by an internal transfer of heat that reduces the magnitude of external heat addition, a feature known as regeneration. It was also seen in Chapter 2 that this is accomplished conveniently in a steam power plant by using a heat exchanger known as a feedwater heater.

Examination of Example 5.1 shows that a similar opportunity exists for the gas turbine cycle. The results show that the combustion process heats the incoming air from 924°F to 1860°F and that the gas turbine exhausts to the atmosphere at 1273°F. Thus a maximum temperature potential of 1273 – 924 = 349°F exists for heat transfer. As in the Rankine cycle, this potential for regeneration can be exploited by incorporation of a heat exchanger. Figure 5.6 shows a gas turbine with a counterflow heat exchanger that extracts heat from the turbine exhaust gas to preheat the compressor discharge air to \(T_c\) ahead of the combustor. As a result, the temperature rise in the combustor is reduced to \(T_3 - T_c\), a reduction reflected in a direct decrease in fuel consumed.

Note that the compressor and turbine inlet and exit states can be the same as for a simple cycle. In this case the compressor, turbine, and net work as well as the work ratio are unchanged by incorporating a heat exchanger.

The effectiveness of the heat exchanger, or regenerator, is a measure of how well it uses the available temperature potential to raise the temperature of the compressor discharge air. Specifically, it is the actual rate of heat transferred to the air divided by the maximum possible heat transfer rate that would exist if the heat exchanger had infinite heat transfer surface area. The actual heat transfer rate to the air is \(mc_p(T_c - T_2)\), and the maximum possible rate is \(mc_p(T_4 - T_2)\). Thus the regenerator effectiveness can be written as

\[
y_{\text{reg}} = \frac{(T_c - T_2)}{(T_4 - T_2)} \quad \text{[dl]} \quad (5.26)
\]

and the combustor inlet temperature can be written as

\[
T_c = T_2 + y_{\text{reg}}(T_4 - T_2) \quad \text{[R | K]} \quad (5.27)
\]

It is seen that the combustor inlet temperature varies from \(T_2\) to \(T_4\) as the regenerator effectiveness varies from 0 to 1. The regenerator effectiveness increases as its heat transfer area increases. Increased heat transfer area allows the cold fluid to absorb more heat from the hot fluid and therefore leave the exchanger with a higher \(T_c\).

On the other hand, increased heat transfer area implies increased pressure losses on both air and gas sides of the heat exchanger, which in turn reduces the turbine pressure ratio and therefore the turbine work. Thus, increased regenerator effectiveness implies a tradeoff, not only with pressure losses but with increased heat exchanger size and complexity and, therefore, increased cost.
The exhaust gas temperature at the exit of the heat exchanger may be determined by applying the steady-flow energy equation to the regenerator. Assuming that the heat exchanger is adiabatic and that the mass flow of fuel is negligible compared with the air flow, and noting that no shaft work is involved, we may write the steady-flow energy equation for two inlets and two exits as

$$ q = 0 = h_e + h_c - h_2 - h_4 + w = c_{p,g}T_e + c_pT_c - c_pT_2 - c_{p,g}T_4 + 0 $$

Thus the regenerator combustion-gas-side exit temperature is:

$$ T_e = T_4 - (c_p/c_{p,g})(T_c - T_2) \quad [R \, K] \quad (5.28) $$

While the regenerator effectiveness does not appear explicitly in Equation (5.28), the engine exhaust temperature is reduced in proportion to the air temperature rise in the regenerator, which is in turn proportional to the effectiveness. The dependence of the exhaust temperature on $y_{\text{reg}}$ may be seen directly by eliminating $T_c$ from Equation (5.28), using Equation (5.27) to obtain

$$ T_4 - T_e = y_{\text{reg}} (c_p/c_{p,g})(T_4 - T_2) \quad [R \, K] \quad (5.29) $$
The regenerator exhaust gas temperature reduction, $T_4 - T_e$, is seen to be jointly proportional to the effectiveness and to the maximum temperature potential, $T_4 - T_2$.

The regenerator, like other heat exchangers, is designed to have minimal pressure losses on both air and gas sides. These may be taken into account by the fractional pressure drop approach discussed in connection with the combustor.

**EXAMPLE 5.2**

Let’s say we are adding a heat exchanger with an effectiveness of 75% to the engine studied in Example 5.1. Assume that the same frictional pressure loss factor applies to both the heat exchanger air-side and combustor as a unit, and that gas-side pressure loss in the heat exchanger is negligible. Evaluate the performance of the modified engine.

Solution

The solution in spreadsheet format, expressed in terms of the notation of Figures 5.3 and 5.7, is shown in Table 5.2. Examination of the spreadsheet and of the $T$-$s$ diagram in Figure 5.7 shows that the entry and exit states of the turbomachines are not influenced by the addition of the heat exchanger, as expected. (There would have been a slight influence if a different pressure loss model had been assumed.)

With the heat exchanger, it is seen that the combustor inlet temperature has increased about 262° and the exhaust temperature reduced 229°. The net work and
work ratio are clearly unchanged. Most importantly, however, the thermal efficiency has increased 10 percentage counts over the simple cycle case in Example 5.1. Such a gain must be traded off against the added volume, weight, and expense of the regenerator. The efficiency gain and the associated penalties may be acceptable in stationary power and ground and marine transportation applications, but are seldom feasible in aerospace applications. Each case, of course, must be judged on its own merits.

Figure 5.8 shows the influence of regenerator effectiveness and turbine inlet temperature on the performance of the gas turbine, all other conditions being the same as in the example. The values for $y_{\text{reg}} = 0$ correspond to a gas turbine without regenerator. The abscissa is arbitrarily truncated at $y_{\text{reg}} = 0.8$ because gas turbine heat exchanger effectivenesses usually do not exceed that value. The impressive influence of both design parameters is a strong motivator for research in heat exchangers and high-temperature materials. The use of regeneration in automotive gas turbines is virtually mandated because good fuel economy is so important.
Figure 5.9 shows the layout of a regenerative gas turbine serving a pipeline compressor station. Gas drawn from the pipeline may be used to provide the fuel for remotely located gas-turbine-powered compressor stations. (A later figure, Figure 5.12, shows details of the turbomachinery of this gas turbine.)
Two-Shaft Gas Turbines

Problems in the design of turbomachinery for gas turbines and in poor part-load or off-design performance are sometimes avoided by employing a two-shaft gas turbine, in which the compressor is driven by one turbine and the load by a second turbine. Both shafts may be contained in a single structure, or the turbines may be separately packaged. Figure 5.10 shows the flow and T-s diagrams for such a configuration. The turbine that drives the compressor is called the compressor turbine. The compressor, combustor, compressor-turbine combination is called the gas generator, or gasifier, because its function is to provide hot, high-pressure gas to drive the second turbine, the power turbine. The compressor-turbine is sometimes also referred to as the gasifier turbine or gas-generator turbine.

The analysis of the two-shaft gas turbine is similar to that of the single shaft machine, except in the determination of the turbine pressure ratios. The pressure rise produced by the compressor must be shared between the two turbines. The manner in which it is shared is determined by a power, or work, condition. The work condition expresses mathematically the fact that the work produced by the gasifier turbine is used to drive the compressor alone. As a result, the gas generator turbine pressure ratio, \( p_3/p_4 \), is just high enough to satisfy the compressor work requirement.

Thus the compressor power (work) input is the same as the delivered gas-generator turbine power(work) output:

\[
|w_c| = y_{mech} (1 + f) w_t \quad \text{[Btu/lbm | kJ/kg]} \quad (5.30)
\]

where \( f \) is the fuel–air ratio and \( y_{mech} \) is the mechanical efficiency of transmission of power from the turbine to the compressor. The mechanical efficiency is usually close to unity in a well-designed gas turbine. For this reason, it was not included in earlier analyses.

The gasifier turbine work may be written in terms of the turbine pressure ratio:

\[
w_t = y_t \ c_{p,g} T_3 (1 - T_{4s}/T_3)
\]

\[
= y_t \ c_{p,g} T_3 [1 - 1/(p_3/p_4)]^{(kg-1)/kg} \quad \text{[Btu/lbm | kJ/kg]} \quad (5.31)
\]

With the compressor work determined, as before, by the compressor pressure ratio and the isentropic efficiency, the compressor-turbine pressure ratio, \( p_3/p_4 \), is obtained by combining Equations (5.30) and (5.31):

\[
p_3/p_4 = \left[ 1 - |w_c|/y_{mech} y_t c_{p,g} (1 + f) T_3 \right]^{-kg/(kg-1)}
\]

\[
= (1 - wf)^{-kg/(kg-1)} \quad \text{[dl]} \quad (5.32)
\]

where \( wf \) is the positive, dimensionless work factor, \(|w_c|/y_{mech} y_t c_{p,g} (1 + f) T_3\), used as
a convenient intermediate variable. The power turbine pressure ratio may then be determined from the identity
\[ \frac{p_4}{p_5} = \frac{p_4}{p_1} = \frac{p_4}{p_3}(\frac{p_3}{p_2})(\frac{p_2}{p_1}). \]
This shows that the power turbine pressure ratio is the compressor pressure ratio divided by the gasifier turbine pressure ratio when there is no combustion chamber pressure loss (\( p_3 = p_2 \)). With the pressure ratios known, all the significant temperatures and performance parameters may be
determined.

**EXAMPLE 5.3**

Let’s consider a two-shaft gas turbine with a regenerative air heater. The compressor pressure ratio is 6, and the compressor and gas generator turbine inlet temperatures are 520 °R and 1860°R, respectively. The compressor, gasifier turbine, and power turbine isentropic efficiencies are 0.86, 0.89, and 0.89, respectively. The regenerator effectiveness is 75%, and a 4% pressure loss is shared by the high-pressure air side of the regenerator and the combustor. Determine the pressure ratios of the two turbines, and the net work, thermal efficiency, and work ratio of the engine.
The solution in spreadsheet form shown in Table 5.3 follows the notation of Figure 5.10. The solution proceeds as before, until the calculation of the turbine pressure ratios. The available pressure ratio shared by the two turbines is $p_3/p_5 = p_3/p_1 = (p_2/p_1)(p_3/p_2) = r (1 - f_{pl}) = 5.76$. The gasifier turbine pressure ratio is determined by the work-matching requirement of the compressor and its driving turbine, as expressed in Equation (5.32), using the dimensionless compressor work factor, $w_f$. The resulting gas generator and power turbine pressure ratios are 2.61 and 2.2, respectively.

Comparison shows that the design point performance of the two-shaft gas turbine studied here is not significantly different from that of the single-shaft machine considered an Example 5.2. While the performance of the two machines is found to be essentially the same, the single-shaft machine is sometimes preferred in applications.
with fixed operating conditions where good part-load performance over a range of speeds is not important. On the other hand, the independence of the speeds of the gas generator and power turbine in the two-shaft engine allows acceptable performance over a wider range of operating conditions.

Let us examine further the characteristics of regenerative two-shaft gas turbines, starting with the spreadsheet reproduced in Table 5.3. By copying the value column of that spreadsheet to several columns to the right, a family of calculations with identical methodologies may be performed. The spreadsheet /EDIT-FILL command may then be used to vary a parameter in a given row by creating a sequence of numbers with a specified starting value and interval. Such a parametric study of the influence of compressor pressure ratio on two-shaft regenerative gas turbine performance is shown in Table 5.4, where the pressure ratio is varied from 2 to 7. The fifth numeric column contains the values from Table 5.3. The data of Table 5.4 are included in the Example.

### Table 5.4 Spreadsheet 5-4.WK1

<table>
<thead>
<tr>
<th>Two-shaft regenerative gas turbine comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
</tr>
<tr>
<td>(k-1)/k = e,a</td>
</tr>
<tr>
<td>T1,R</td>
</tr>
<tr>
<td>etac</td>
</tr>
<tr>
<td>etatg</td>
</tr>
<tr>
<td>etatp</td>
</tr>
<tr>
<td>r</td>
</tr>
<tr>
<td>T2s,R</td>
</tr>
<tr>
<td>T2,R</td>
</tr>
<tr>
<td>Wc,Btu/lb</td>
</tr>
<tr>
<td>fpl</td>
</tr>
<tr>
<td>p3/p1</td>
</tr>
<tr>
<td>T3,R</td>
</tr>
<tr>
<td>kg</td>
</tr>
<tr>
<td>(kg-1)/kg=e,g</td>
</tr>
<tr>
<td>cp.g,Btu/lb-R</td>
</tr>
<tr>
<td>work factor</td>
</tr>
<tr>
<td>rtg</td>
</tr>
<tr>
<td>rtp</td>
</tr>
<tr>
<td>T4s,R</td>
</tr>
<tr>
<td>T4,R</td>
</tr>
<tr>
<td>T5s,R</td>
</tr>
<tr>
<td>T5,R</td>
</tr>
<tr>
<td>Wc,Btu/lb</td>
</tr>
<tr>
<td>etareg</td>
</tr>
<tr>
<td>Te,R</td>
</tr>
<tr>
<td>Qc, Btu/lb</td>
</tr>
<tr>
<td>Wn,Btu/lb</td>
</tr>
<tr>
<td>thermal eff.</td>
</tr>
<tr>
<td>work ratio</td>
</tr>
<tr>
<td>1.40</td>
</tr>
<tr>
<td>0.2857</td>
</tr>
<tr>
<td>520.00</td>
</tr>
<tr>
<td>0.86</td>
</tr>
<tr>
<td>0.89</td>
</tr>
<tr>
<td>2.00</td>
</tr>
<tr>
<td>533.89</td>
</tr>
<tr>
<td>652.43</td>
</tr>
<tr>
<td>-31.78</td>
</tr>
<tr>
<td>0.04</td>
</tr>
<tr>
<td>1.92</td>
</tr>
<tr>
<td>1860.00</td>
</tr>
<tr>
<td>1.33</td>
</tr>
<tr>
<td>0.25</td>
</tr>
<tr>
<td>0.2744</td>
</tr>
<tr>
<td>-0.07</td>
</tr>
<tr>
<td>1.34</td>
</tr>
<tr>
<td>1.44</td>
</tr>
<tr>
<td>1729.86</td>
</tr>
<tr>
<td>1744.17</td>
</tr>
<tr>
<td>1593.19</td>
</tr>
<tr>
<td>1609.80</td>
</tr>
<tr>
<td>38.87</td>
</tr>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>1370.45</td>
</tr>
<tr>
<td>981.78</td>
</tr>
<tr>
<td>134.33</td>
</tr>
<tr>
<td>36.87</td>
</tr>
<tr>
<td>0.27</td>
</tr>
<tr>
<td>2.16</td>
</tr>
<tr>
<td>1.40</td>
</tr>
<tr>
<td>0.2857</td>
</tr>
<tr>
<td>520.00</td>
</tr>
<tr>
<td>0.86</td>
</tr>
<tr>
<td>0.89</td>
</tr>
<tr>
<td>4.00</td>
</tr>
<tr>
<td>711.74</td>
</tr>
<tr>
<td>742.96</td>
</tr>
<tr>
<td>-53.51</td>
</tr>
<tr>
<td>0.04</td>
</tr>
<tr>
<td>2.88</td>
</tr>
<tr>
<td>1860.00</td>
</tr>
<tr>
<td>1.33</td>
</tr>
<tr>
<td>0.25</td>
</tr>
<tr>
<td>0.2744</td>
</tr>
<tr>
<td>-0.12</td>
</tr>
<tr>
<td>1.65</td>
</tr>
<tr>
<td>1.74</td>
</tr>
<tr>
<td>1571.22</td>
</tr>
<tr>
<td>1664.99</td>
</tr>
<tr>
<td>1448.76</td>
</tr>
<tr>
<td>1472.55</td>
</tr>
<tr>
<td>60.42</td>
</tr>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>1240.56</td>
</tr>
<tr>
<td>1602.98</td>
</tr>
<tr>
<td>1355.57</td>
</tr>
<tr>
<td>1382.79</td>
</tr>
<tr>
<td>64.21</td>
</tr>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>1206.18</td>
</tr>
<tr>
<td>1551.25</td>
</tr>
<tr>
<td>1288.31</td>
</tr>
<tr>
<td>1317.23</td>
</tr>
<tr>
<td>65.93</td>
</tr>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>1180.69</td>
</tr>
<tr>
<td>1506.46</td>
</tr>
<tr>
<td>1236.49</td>
</tr>
<tr>
<td>1268.19</td>
</tr>
<tr>
<td>66.40</td>
</tr>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>1160.97</td>
</tr>
<tr>
<td>1025.83</td>
</tr>
<tr>
<td>1041.86</td>
</tr>
<tr>
<td>191.81</td>
</tr>
<tr>
<td>66.40</td>
</tr>
<tr>
<td>0.35</td>
</tr>
<tr>
<td>1.86</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>0.63</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>1.76</td>
</tr>
<tr>
<td>1.68</td>
</tr>
</tbody>
</table>

Table entries are calculated using the spreadsheet methodology described above.
Table 5.4 shows that, for the given turbine inlet temperature, the thermal efficiency maximum is at a pressure ratio between 4 and 5, while the net work maximum is at a pressure ratio of about 7. The work ratio is continually declining because the magnitude of the compressor work requirement grows faster with compressor pressure ratio than the turbine work does.

Figure 5.11, plotted using the spreadsheet, compares the performance of the regenerative two-shaft gas turbine with a nonregenerative two-shaft engine ($y_{reg} = 0$). Net work for both machines has the same variation with pressure ratio. But notice the high efficiency attained with a low-compressor pressure ratio, a significant advantage attributable to regeneration.

A cutaway view of a two-shaft regenerative gas turbine of the type used in pipeline compressor stations such as that shown in Figure 5.9 is seen in Figure 5.12. The figure shows that the compressor blade heights decrease in the direction of flow as the gas is compressed. The exhaust from the last of the sixteen compressor stages is reduced in velocity by a diffusing passage and then exits through the right window-like flange, which connects to a duct (not shown) leading to the regenerator. The heated air from the external regenerator reenters the machine combustor casing, where it flows around and into the combustor cans, cooling them. The air entering near the combustor fuel nozzles mixes with the fuel and burns locally in a near-stoichiometric mixture. As the mixture flows downstream, additional secondary air entering the combustor through slots in its sides mixes with, and reduces the temperature of, the combustion gas before it arrives at the turbine inlet.

A cutaway view of an industrial two-shaft gas turbine with dual regenerators is presented in Figure 5.13. From the left, the air inlet and radial compressor and axial flow gasifier turbine and power turbine are seen on the axis of the machine, with the combustion chamber above and one of the rotary regenerators at the right. Due to the relatively low pressure ratios required by regenerative cycles, centrifugal compressors are normally used in regenerative machines because of their simplicity, good efficiency, compactness, and ruggedness.

Performance data for the turbine of Figure 5.13 is graphed in Figure 5.14. The GT 404 gas turbine delivers about 360 brake horsepower at 2880-rpm output shaft speed. The torque-speed curve of Figure 5.14 shows an important characteristic of two-shaft gas turbines with respect to off-design point operation. Whereas the compressor pressure ratio and output torque of a single-shaft gas turbine drop as the shaft speed drops, the compressor speed and pressure ratio in a two-shaft machine is independent of the output speed. Thus, as the output shaft speed changes, the compressor may maintain its design speed and continue to develop high pressure and mass flow. Thus the torque at full stall of the output shaft of the GT404 is more than twice the full-load design torque. This high stall torque is superior to that of reciprocating engines and is important in starting and accelerating rotating equipment that has high initial turning resistance. This kind of engine may be used in truck, bus, and marine applications as well as in an industrial setting.
FIGURE 5.11 Performance of two-shaft gas turbines.

FIGURE 5.12 Two-shaft regenerative combustion turbine. (Courtesy of Westinghouse Canada.)
FIGURE 5.13 GT404 regenerative industrial two-shaft gas turbine. (Courtesy of the Allison Gas Turbine Division, General Motors Corporation.)

GT404 Performance

ISO
Zero Inlet/Exhaust Losses
(Full Throttle)

FIGURE 5.14 Torque and power characteristics of the GT404 gas turbine. (Courtesy of the Allison Gas Turbine Division, General Motors Corporation.)
A unique patented feature of some of the Allison gas turbines, called “power transfer,” is the ability to link the dual shafts. A hydraulic clutch mechanism between the two turbine shafts acts to equalize their speeds. This tends to improve part-load fuel economy, and provides engine braking and overspeed protection for the power turbine. When the clutch mechanism is fully engaged, the shafts rotate together as a single-shaft machine.

**Intercooling and Reheat Intercooling**

It has been pointed out that the work of compression extracts a high toll on the output of the gas turbine. The convergence of lines of constant pressure on a $T$-$s$ diagram indicates that compression at low temperatures reduces compression work. The ideal compression process would occur isothermally at the lowest available temperature. Isothermal compression is difficult to execute in practice. The use of multistage compression with intercooling is a move in that direction.

Consider replacing the isentropic single-stage compression from $p_1$ to $p_2 = p_2^*$ in Figure 5.15 with two isentropic stages from $p_1$ to $p_{iS}$ and $p_{i^*}$ to $p_{2S}$. Separation of the compression processes with a heat exchanger that cools the air at $T_{iS}$ to a lower temperature $T_{i^*}$ acts to move the final compression process to the left on the $T$-$s$
diagram and reduces the discharge temperature following compression to $T_{2s}$. A heat exchanger used to cool compressed gas between stages of compression is called an intercooler.

The work required to compress from $p_1$ to $p_{2s} = p_2^* = p_2$ in two stages is

$$w_c = c_p [(T_1 - T_{is}) + (T_1^* - T_{2s})] \quad \text{[Btu/lbm | kJ/kg]}$$

Note that intercooling increases the net work of the reversible cycle by the area $is-i^*–2s–2^*–is$. The reduction in the work due to two-stage intercooled compression is also given by this area. Thus intercooling may be used to reduce the work of compression between two given pressures in any application. However, the favorable effect on compressor work reduction due to intercooling in the gas turbine application may be offset by the obvious increase in combustor heat addition, $c_p (T_2^* - T_{2s})$, and by increased cost of compression system. The next example considers the selection of the optimum pressure level for intercooling, $p_i = p_{is} = p_i^*$.

**EXAMPLE 5.4**

Express the compressor work, for two-stage compression with intercooling back to the original inlet temperature, in terms of compressor efficiencies and pressure ratios.

Develop relations for the compressor pressure ratios that minimize the total work of compression in terms of the overall pressure ratio.

Solution

Taking $T_i^* = T_1$ as directed in the problem statement, and letting $r = p_2/p_1$, $r_1 = p_{is}/p_1$ and $r_2 = p_{2s}/p_i^* = r/r_1$ as in Figure 5.15, we get for the compression work,

$$w_c = c_p [(T_1 - T_{is})/y_c1 + (T_1 - T_{2s})/y_c2]$$

$$= c_p T_1[(1 - T_{is}/T_1)/y_c1 + (1 - T_{2s}/T_1)/y_c2]$$

$$= c_p T_1[(1 - r_1 (k - 1)/k )/y_c1 + (1 - r_2 (k - 1)/k )/y_c2] \quad \text{[Btu/lbm | kJ/kg]}$$

Eliminating $r_2$, using $r = r_1 r_2$, yields

$$w_c = c_p T_1[(1 - r_1 (k - 1)/k )/y_c1 + (1 - (r/r_1) (k - 1)/k )/y_c2]$$

Differentiating with respect to $r_1$ for a fixed $r$ and setting the result equal to zero, we obtain

$$-r_1^{-1/k} l_i^{+} (r (k - 1)/k) l_g^{-} (2k - 1)/k = 0$$

which simplifies to
Using this result we find also that

\[ r_{2\text{opt}} = \left( \frac{y_{c2}}{y_{c1}} \right)^{k/(2k-1)} r^{1/2} \]

Examination of these equations shows that, for compressors with equal efficiencies, both compressor stages have the same pressure ratio, which is given by the square root of the overall pressure ratio. For unequal compressor efficiencies, the compressor with the higher efficiency should have the higher pressure ratio.

---

**Reheat**

Let us now consider an improvement at the high-temperature end of the cycle. Figure 5.16 shows the replacement of a single turbine by two turbines in series, each with appropriately lower pressure ratios, and separated by a reheater. The *reheater* may be a combustion chamber in which the excess oxygen in the combustion gas leaving the first turbine burns additional fuel, or it may be a heater in which external combustion provides the heat necessary to raise the temperature of the working fluid to \( T_{m*} \). The high temperature at the low-pressure turbine inlet has the effect of increasing the area of the cycle by \( m - m^* - 4 - 4^* m \) and hence of increasing the net work.
Like intercooling, the increase in net work is made possible by the spreading of the constant pressure lines on the T-s diagram as entropy increases. Thus the increase in turbine work is

$$A_w = c_{p,g} \left[ (T_m^* - T_4) - (T_m - T_4^*) \right] \quad [\text{Btu/lbm} | \text{kJ/kg}]$$

(5.33)

Also as with intercooling, the favorable effect in increasing net work is offset by the reduction of cycle efficiency resulting from increased addition of external heat from the reheater:

$$q_{rh} = c_{p,g} (T_m^* - T_m) \quad [\text{Btu/lbm} | \text{kJ/kg}]$$

(5.34)

As with intercooling, the question arises as to how the intermediate pressure for reheat will be selected. An analysis similar to that of Example 5.4 shows the unsurprising result that the reheat pressure level should be selected so that both turbines have the same expansion ratio if they have the same efficiencies and the same inlet temperatures.

**Combining Intercooling, Reheat, and Regeneration**

Because of their unfavorable effects on thermal efficiency, intercooling and reheat alone or in combination are unlikely to be found in a gas turbine without another feature that has already been shown to have a favorable influence on gas turbine fuel economy: a regenerator. The recuperator or regenerator turns disadvantage into advantage in a cycle involving intercooling and/or reheat. Consider the cycle of Figure 5.17, which incorporates all three features.

The increased turbine discharge temperature $T_4$ produced by reheat and the decreased compressor exit temperature $T_2$ due to intercooling both provide an enlarged temperature potential for regenerative heat transfer. Thus the heat transfer $c_p (T_c - T_2)$ is accomplished by an internal transfer of heat from low pressure turbine exhaust gas. This also has the favorable effect of reducing the temperature of the gas discharged to the atmosphere. The requisite external heat addition for this engine is then

$$q_a = c_p \left[ (T_3 - T_c) + (T_m^* - T_m) \right] \quad [\text{Btu/lbm} | \text{kJ/kg}]$$

(5.35)

Thus the combination of intercooling, reheat, and regeneration has the net effect of raising the average temperature of heat addition and lowering the average temperature of heat rejection, as prescribed by Carnot for an efficient heat engine.
The Ericsson cycle

Increasing the number of intercoolers and reheaters without changing the overall pressure ratio may be seen to cause both the overall compression and the overall expansion to approach isothermal processes. The resulting reversible limiting cycle, consisting of two isotherms and two isobars, is called the Ericsson cycle. With perfect internal heat
transfer between isobaric processes, all external heat addition would be at the maximum temperature of the cycle and all heat rejected at the lowest temperature. Analysis of the limiting reversible cycle reveals, as one might expect, that its efficiency is that of the Carnot cycle. Plants with multistage compression, reheat, and regeneration can have high efficiencies; but complexity and high capital costs have resulted in few plants that actually incorporate all these features.
Gas turbines are used in aircraft to produce shaft power and hot, high-pressure gas for jet propulsion. Turbine shaft power is used in turboprop aircraft and helicopters to drive propellers and rotors. A modern turboprop engine and an aircraft that uses it are shown in Figures 5.18 through 5.20. While its jet exhaust provides some thrust, the bulk of the propulsive thrust of the turboprop is provided by its propeller. The rear-
ward acceleration of a large air mass by the propeller is responsible for the good fuel economy of turboprop aircraft. Thus the turboprop is popular as a power plant for small business aircraft. At higher subsonic flight speeds, the conventional propeller loses efficiency and the turbojet becomes superior.

Auxiliary power units, APUs, are compact gas turbines that provide mechanical power to generate electricity in transport aircraft while on the ground. The thermo-dynamic fundamentals of these shaft-power devices are the same those of stationary gas turbines, discussed earlier. Their design, however, places a premium on low weight and volume and conformance to other constraints associated with airborne equipment. Thus their configuration and appearance may differ substantially from those of other stationary gas turbines.

The jet engine consists of a gas turbine that produces hot, high-pressure gas but has zero net shaft output. It is a gasifier. A nozzle converts the thermal energy of the hot, high-pressure gas produced by the turbine into a high-kinetic-energy exhaust stream. The high momentum and high exit pressure of the exhaust stream result in a forward thrust on the engine.

Although the analysis of the jet engine is similar to that of the gas turbine, the configuration and design of jet engines differ significantly from those of most stationary gas turbines. The criteria of light weight and small volume, mentioned earlier, apply here as well. To this we can add the necessity of small frontal area to minimize the aerodynamic drag of the engine, the importance of admitting air into the
engine as efficiently (with as little stagnation pressure loss) as possible, and the efficient conversion of high-
temperature turbine exit gas to a high-velocity nozzle exhaust. The resulting configuration is shown schematically in
Figure 5.21.

Up to now we have not been concerned with kinetic energy in the flows in gas turbines, because the flows at
the stations of interest are usually designed to have low velocities. In the jet engine, however, high kinetic energy is
present in the free stream ahead of the engine and in the nozzle exit flow. The analysis here will therefore be
presented in terms of stagnation, or total, temperatures and pressures, where kinetic
energy is taken into account implicitly, as discussed in Section 1.7. The preceding analyses may be readily adapted to deal with the stagnation properties associated with compressible flow. In the following discussion, engine processes are first described and then analyzed.

It should be recalled that if there are no losses, as in an isentropic flow, the stagnation pressure of a flow remains constant. All loss mechanisms, such as fluid friction, turbulence, and flow separation, decrease stagnation pressure. Only by doing work on the flow (with a compressor, for example) is it possible to increase stagnation pressure.

In Figure 5.21, free-stream ambient air, denoted by subscript a, enters an engine inlet that is carefully designed to efficiently decelerate the air captured by its frontal area to a speed low enough to enter the compressor, at station 1, with minimal aerodynamic loss. There is stagnation pressure loss in the inlet, but efficient deceleration of the flow produces static and stagnation pressures at the compressor entrance well above the ambient free-stream static pressure. This conversion of relative kinetic energy of ambient air to increased pressure and temperature in the engine inlet is sometimes called ram effect.

The compressor raises the stagnation pressure of the air further to its maximum value at station 2, using power delivered by the turbine. Fuel enters the combustion chamber and is burned with much excess air to produce the high turbine inlet temperature at station 3. We adopt here, for simplicity, the familiar idealization that no pressure losses occur in the combustion chamber. The hot gases then expand through the turbine and deliver just enough power to drive the compressor (the work condition again). The gases leave the turbine exit at station 4, still hot and at a stagnation pressure sure well above the ambient. These gases then expand through a nozzle that converts the excess pressure and thermal energy into a high-kinetic-energy jet at station 5. The forward thrust on the engine, according to Newton’s Second Law, is produced by the reaction to the internal forces that accelerate the internal flow rearward to a high jet velocity and the excess of the nozzle exit plane pressure over the upstream ambient pressure.

Inlet Analysis

Given the flight speed, \( V_a \), and the free-stream static temperature and pressure, \( T_a \) and \( p_a \), at a given altitude, the free-stream stagnation temperature and pressure are

\[
T_{oa} = T_a + \frac{V_a^2}{2c_p} \quad \text{[R | K]} \tag{5.36}
\]

and

\[
p_{oa} = p_a \left( \frac{T_{oa}}{T_a} \right)^{k/(k-1)} \quad \text{[lb f/ft}^2 \ | \text{kPa]} \tag{5.37}
\]

Applying the steady-flow energy equation to the streamtube entering the inlet, we find
that the stagnation enthalpy \( h_{oa} = h_{o1} \) for adiabatic flow. For subsonic flight and supersonic flight at Mach numbers near one, the heat capacity of the air is essentially constant. Thus constancy of the stagnation enthalpy implies constancy of the stagnation temperature. Hence, using Equation (5.36),

\[
T_{o1} = T_{oa} = T_a + V_a^2 / 2c_p \quad [\text{R} | \text{K}] \tag{5.38}
\]

The effects of friction, turbulence, and other irreversibilities in the inlet flow are represented by the inlet pressure recovery, \( \text{PR} \), defined as

\[
\text{PR} = p_{o1} / p_{oa} \quad [\text{dl}] \tag{5.39}
\]

where an isentropic flow through the inlet has a pressure recovery of 1.0. Lower values indicate reduced inlet efficiency and greater losses. For subsonic flow, values on the order of 0.9 to 0.98 are typical. At supersonic speeds the pressure recovery decreases with increasing Mach number.

**Compressor Analysis**

With the stagnation conditions known at station 1 in Figure 5.21, the compressor pressure ratio, \( r = p_{o2} / p_{o1} \), now yields \( p_{o2} \); and the isentropic relation, Equation (1.19), gives the isentropic temperature, \( T_{o2s} \):

\[
T_{o2s} = T_{o1}r^{(k-1)/k} \quad [\text{R} | \text{K}] \tag{5.40}
\]

The actual compressor discharge stagnation temperature is then obtained from the definition of the compressor efficiency in terms of stagnation temperatures:

\[
\gamma_{\text{comp}} = (T_{o1} - T_{o2s}) / (T_{o1} - T_{o2}) \quad [\text{dl}] \tag{5.41}
\]

**Combustor and Turbine Analysis**

The turbine inlet temperature, \( T_{03} \), is usually assigned based on turbine blade material considerations. For preliminary analysis it may be assumed that there are negligible pressure losses in the combustion chamber, so that \( p_{03} = p_{02} \). As with the two-shaft gas turbine, the condition that the power absorbed by the compressor equal the power delivered by the turbine determines the turbine exit temperature, \( T_{04} \):

\[
c_p(T_{o2} - T_{o1}) = (1 + f)c_{p,g}T_{o3}(1 - T_{04} / T_{o3}) [\text{Btu} | \text{kJ}] \tag{5.42}
\]

where \( f \) is the engine fuel-air ratio, which often may be neglected with respect to 1 (as in our earlier studies) when high precision is not required.
The turbine efficiency equation then yields the isentropic discharge temperature \( T_{04s} \), and Equation (1.19) yields the turbine pressure ratio:

\[
T_{04s} = T_{03} - (T_{03} - T_{04}) / y_{turb} \quad [R \mid K]
\]

\[
p_{03} / p_{04} = (T_{03} / T_{04s})^{kg/(kg - 1)} \quad [dl]
\]

Thus the stagnation pressure and temperature at station 4 are known. Note that the turbine pressure ratio is usually significantly lower than the compressor pressure ratio.

**Nozzle Analysis**

The flow is then accelerated to the jet velocity at station 5 by a *convergent nozzle* that contracts the flow area. A well-designed nozzle operating at its design condition has only small stagnation pressure losses. Hence the nozzle here is assumed to be loss-free and therefore isentropic.

Under most flight conditions the exhaust nozzle is *choked*; that is, it is passing the maximum flow possible for its upstream conditions. A choked nozzle has the local flow velocity at its minimum area, or *throat*, equal to the local speed of sound. As a result, simple relations exist between the upstream stagnation conditions at station 4 and the choked conditions at the throat. Thus, for a choked isentropic nozzle,

\[
T_{04} = T_{05} = T_5 + V_5^2 / 2c_{p,g}
\]

\[
= T_5 + k_g R T_5 / 2c_{p,g} = T_5 (1 + k_g R / 2c_{p,g})
\]

\[
= T_5 (k_g + 1) / 2 \quad [R \mid K]
\]

(5.45)

where \( k_g R / c_{p,g} = k_g - 1 \). With \( k_g = 1.333 \) for the combustion gas, this determines the exit temperature \( T_5 \).

Combining the isentropic relation with Equation (5.45) then gives the nozzle exit static pressure \( p_5 \):

\[
p_5 / p_{04} = (T_5 / T_{04})^{kg/(kg - 1)}
\]

\[
= [2/(k_g + 1)]^{kg/(kg - 1)} \quad [dl]
\]

(5.46)

**EXAMPLE 5.5**

The stagnation temperature and pressure leaving a turbine and entering a convergent nozzle are 970.2K. and 2.226 bar, respectively. What is the static pressure and temperature downstream if the nozzle is choked? If the free-stream ambient pressure is 0.54 bar, is the nozzle flow choked? Compare the existing nozzle pressure ratio with the critical pressure ratio.
Solution

If the nozzle is choked, then from Equations (5.45) and (5.46),

$$T_5 = 2T_{o4}/(kg + 1) = 2(970.2)/2.333 = 831.6 \text{ K}$$

and the static pressure at the nozzle throat is

$$p_5 = p_{o4} \left[2/(kg + 1)\right]^{kg/(kg - 1)} = 2.226(2/2.333)^4 = 1.202 \text{ bar}$$

The fact that \(p_5 > p_a\) = 0.54 indicates that the nozzle is choked.

The critical pressure ratio of the nozzle is

$$p_{o4}/p_5 = [(kg + 1)/2]^{kg/(kg - 1)} = (2.333/2)^4 = 1.852$$

and the applied pressure ratio is 2.226/0.54 = 4.12. Thus the applied pressure ratio exceeds the critical pressure ratio. This also indicates that the nozzle is choked.

For the isentropic nozzle, the steady-flow energy equation gives

$$0 = h_5 + V_5^2/2 - h$$

or, with \(c_{p,g}\) constant,

$$V_5 = \left[2c_{p,g}(T_{o4} - T_5)\right]^{1/2} \text{ [ft/s | m/s]} \quad (5.47)$$

Thus the jet velocity is determined from Equation (5.47), where \(T_5\) is obtained from Equation (5.45).

The thrust of the engine is obtained by applying Newton’s Second Law to a control volume, as shown in Figure 5.22. If the mass flow rate through the engine is \(m\), the rates of momentum flow into and out of the control volume are \(mV_a\) and \(mV_5\), respectively. The net force exerted by the exit pressure is \((p_5 - p_a)A_5\), where \(A_5\) is the nozzle exit area. Thus, applying Newton’s Second Law to the control volume, we can relate the force exerted by the engine on the gases flowing through, \(F\), and the net exit pressure force to the rate of increase of flow momentum produced by the engine:

$$m(V_5 - V_a) = F - (p_5 - p_a)A_5 \quad \text{[lb | kN]}$$

or

$$F = m(V_5 - V_a) + (p_5 - p_a)A_5 \quad \text{[lb | kN]} \quad (5.48)$$

Here, \(F\) is the engine force acting on the gas throughput. The reaction to this force is the thrust on the engine acting in the direction of flight. Thus the magnitude of the
thrust is given by Equation (5.48). It is the sum of all the pressure force components acting on the inside the engine in the direction of flight.

The exit area, $A_5$, is related to the mass flow rate by

$$m = A_5 q_5 V_5 \quad [\text{lb}_m/\text{s} \mid \text{kg}/\text{s}]$$

where the density at station 5 is obtained from the perfect gas law using $p_5$ and $T_5$ from Equations (5.45) and (5.46). If $A_5$ is known, the mass flow rate through the engine may be determined from Equation (5.49) and the thrust from Equation (5.48).

Another type of nozzle used in high-performance engines and in rocket nozzles is a convergent-divergent nozzle, one in which the flow area first contracts and then increases. It differs from the convergent nozzle in that it can have supersonic flow at the exit. For such a fully expanded, convergent-divergent nozzle operating at its design condition, $p_5 = p_a$, and the engine thrust from Equation (5.48) reduces to $m(V_5 - V_a)$. 

**FIGURE 5.22** Control volume for the determination of engine thrust.