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DEPARTMENT OF CIVIL ENGINEERING

COURSE MATERIAL
CE 6702- PRESTRESSED CONCRETE STRUCTURES

PREPARED BY
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OBJECTIVES:
- To introduce the need for prestressing as well as the methods, types and advantages of prestressing to the students. Students will be introduced to the design of prestressed concrete structures subjected to flexure and shear.

UNIT I INTRODUCTION – THEORY AND BEHAVIOUR

UNIT II DESIGN FOR FLEXURE AND SHEAR
Basic assumptions for calculating flexural stresses – Permissible stresses in steel and concrete as per I.S.1343 Code – Design of sections of Type I and Type II post-tensioned and pre-tensioned beams – Check for strength limit based on I.S. 1343 Code – Layout of cables in post-tensioned beams – Location of wires in pre-tensioned beams – Design for shear based on I.S. 1343 Code.

UNIT III DEFLECTION AND DESIGN OF ANCHORAGE ZONE
Factors influencing deflections – Short term deflections of uncracked members – Prediction of long term deflections due to creep and shrinkage – Check for serviceability limit state of deflection. Determination of anchorage zone stresses in post-tensioned beams by Magnel”s method, Guyon’s method and IS1343 code – design of anchorage zone reinforcement – Check for transfer bond length in pre-tensioned beams.

UNIT IV COMPOSITE BEAMS AND CONTINUOUS BEAMS
Analysis and design of composite beams – Methods of achieving continuity in continuous beams – Analysis for secondary moments – Concordant cable and linear transformation – Calculation of stresses – Principles of design.

UNIT V MISCELLANEOUS STRUCTURES
Design of tension and compression members – Tanks, pipes and poles – Partial prestressing – Definition, methods of achieving partial prestressing, merits and demerits of partial prestressing.

OUTCOMES:
- Student shall have a knowledge on methods of prestressing and able to design various prestressed concrete structural elements.

TEXT BOOKS:

REFERENCES:
Prestressed concrete – Basic concepts.
The tensile strength of plain concrete is only a fraction of its compressive strength and the problem of it being deficient in tensile strength appers to have the driving factor in the development of the composite material known as Reinforced concrete.

The development of early strength and cracks in reinforced concrete due to incompatibility in the strains of steel and concrete perhaps the starting point in the development of a new material like “Prestressed concrete”.

Definition:
Prestressed concrete is defined as the application of permanent compressive stress to a material like concrete, which is strong in compression but weak in tension, increases the apparent tensile strength of that material, because the subsequent application of tensile stress must first nullify the compressive prestress.

Advantages of prestressed concrete.
✓ In case of fully prestressed member, which are free from tensile stresses under working loads, the cross section is more efficiently utilized when compared with a reinforced concrete section which is cracked under working loads.

✓ The flexural member is stiffer under working loads than a reinforced concrete member of the same length.
✓ Use of high strength concrete and high strength steel provides light and slender member.

Pre tensioning: A method of pre stressing concrete in which the tendons are tensioned before the concrete is placed. In this method, the prestress is imparted to concrete by bond between steel and concrete.

Post tensioning: A method of pre stressing concrete by tensioning the tendons against hardened concrete. In this method, the prestress is imparted to concrete by bearing.

Materials used,
High tensile steel needed for prestressed concrete construction
High strength concrete is necessary for prestress concrete as the material offers highly resistance in tension, shear bond and bearing. Tensile strength of high tensile steel is in the range of 1400 to 2000 N/mm² and if initially stress upto 1400 N/mm² their will be still large stress in the high tensile reinforcement after making deduction for loss of prestress. Therefore high tensile steel is made for prestress concrete.

High strength concrete is needed ..more than 1200N/mm².
Systems of prestressing
✓ Freyssinet systems
✓ Gifford Udall
✓ Lee-Mccall systems
✓ Magnel blaton systems

Methods of prestressing
1. Conventional methods
✓ Circular prestressing
✓ Linear prestressing
✓ Pretensioned method
✓ Post tensioned method

Non Conventional methods
✓ Externally prestressed
✓ Electro – thermal prestressing.
✓ Chemical prestressing.
✓ Prestressing with fibre composites.

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Classifications of prestressed concrete structures
✓ Externally or internally prestressed
✓ Pretensioning and post tensioning
✓ End-Anchored or Non-End Anchored Tendons
✓ Bonded or unbonded tendons
✓ Precast, cast-in-place, composite construction
✓ Partial or full prestressing

Load balancing concept.
It is possible to select cable profiles in a prestressed concrete member such that the traverse component of the cable force balances the given type of external loads. This can be readily illustrated by considering the free body of concrete with the tendon replaced by forces acting on the concrete beam.

Effect on tendon profile on deflections.
In most of the cases of prestressed beams, tendons are located with eccentricities towards the soffit of beams to counteract the sagging bending moments due to transverse loads. Consequently, the concrete beam deflects upwards on the application or transfer of prestress. Since the bending moment at every section is the product of the prestressing force and eccentricity, the tendon profile itself will represent the shape of the BMD.
Tendon profile that affects factors influencing deflections

- Imposed load and self weight
- Magnitude of the prestressing force
- Cable profile
- Span of the member
6) Parabolic and Straight Tendon:

\[ \delta = \frac{P e}{E I} \left[ 5l_1^2 + 12l_1 l_2 + 6l_2^2 \right] \]

7) Parabolic and Straight Tendon (Eccentric Anchors):

\[ \delta = \frac{P(e_1 + e_2)}{12EI} \left[ 5l_1^2 + 12l_1 l_2 + 6l_2^2 \right] + \frac{P e_2 L^2}{8EI} \]

8) Deflection due to Self-weight and Imposed Loads:

\[ \delta = \frac{5(q+q') L^4}{384EI} \]

Long-Term Deflection:

\[ \delta = \left[ (\delta_{il} - \delta_{ip}) \frac{P_t}{P_i} \right] (1+\eta) \]

Where,

- \( P_i \): Initial Prestressing Force
- \( P_t \): Prestressing Force after a time \( t' \)
- \( \delta_{il} \): Initial deflection due to transverse loads
- \( \delta_{ip} \): Initial deflection due to prestress
- \( \eta \): Creep Co-efficient
Short Term Deflection:

Short Term deflection occurs immediately upon the application of a load. It is due to the prestressing force and self-weight.

Long Term Deflection:

Long Term deflection under service loads due to the effective prestressing force and the gravity loads. It takes into account the long term shrinkage and creep movements.

Factors Influencing Deflections:

Factors influencing deflections are:

- Self-weight and imposed load
- Magnitude of the prestressing force
- Cable Profile
- Modulus of Elasticity of Concrete
- Span of the member
- Flexibility Conditions
- Second moment of area of cross-section
- Shrinkage, creep and relaxation of steel stresses.
loss of prestress.

✓ loss due to elastic deformation.
✓ loss due to shrinkage of concrete.
✓ loss due to creep of concrete.
✓ loss due to anchorage slip.
✓ loss due to friction.
✓ loss due to relaxation of stress in steel.

Loss of Prestress:

The initial prestress in concrete undergoes a gradual reduction with time from the stage of tension due to various causes. This is generally referred to as "loss of prestress." A reasonably good estimate of the magnitude of loss of prestress is necessary from the point of view of design.

Types of Losses of Prestress:

Pre-Tensioning

Post-Tensioning

1) Elastic Deformation of Concrete

1) No loss due to elastic deformation if all the wires are simultaneously tensioned.

If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete.
Crack Width in Prestressed Members:

The width of crack that develops in prestressed member is governed by:

* the average strain at the level at which cracks are considered,
* the minimum cover to the tension steel,
* the overall depth of the member,
* the neutral axis depth.

\[ w_{cr} = \frac{3ac_{r}Em}{h_{2} \left[ \frac{ac_{r} - C_{min}}{h-x} \right]} \]

Where,

- \( ac_{r} \) - distance from the point considered to the surface of the nearest longitudinal bar.
- \( E_{m} \) - average strain at the level where cracking is being considered.
- \( C_{min} \) - minimum cover to the tension steel.
- \( h \) - overall depth of the member.
- \( x \) - neutral axis depth.

\[ E_{m} = E_{1} \frac{bt (h-x)(a'-x)}{3EsA_{s}(d-x)} \]

Where,

- \( E_{1} \) - strain at the level considered.
- \( bt \) - width of the section at the centroid of the tension steel.
- \( a' \) - distance from the compression face to the point at which the crack is being calculated.
A rectangular concrete beam 100 mm wide by 250 mm deep, spanning over 8 m, is prestressed by a straight cable carrying an effective prestressing force of 250 kN located at an eccentricity of 40 mm. The beam supports a live load of 1.2 kN/m.

(a) Calculate the resultant stress distribution for the centre of span c/c of the beam assuming the density of concrete as 24 kN/m³.

(b) Find the magnitude of the prestressing force with an eccentricity of 40 mm which can balance the stress due to live load at the sofit of centre span section. [Nov/Dec - 2012]

**Given:**
- \( b = 100 \text{ mm} \)
- \( d = 250 \text{ mm} \)
- \( p = 250 \text{ kN} \)
- \( e = 40 \text{ mm} \)
- \( L = 1.2 \text{ kN/m} \)
- \( f' = 24 \text{ kN/m}^3 \)
- \( L = 8.0 \text{ m} \)

**Solution:**

(i) **Cross Area**
\[
A = 100 \times 250 = 25 \times 10^3 \text{ mm}^2.
\]

**Self. wt. of beam**
\[
g = 0.1 \times 0.25 \times 24
\]
\[
g = 0.6 \text{ kN/m}.
\]

**Total load**
\[
w = g + q
\]
\[
w = 0.6 + 1.2 = 1.8 \text{ kN/m}.
\]

(ii) **Section modulus**
\[
z = \frac{bd^2}{B}
\]
\[
z = \frac{100 \times 250^2}{6}
\]
\[
z = 1.05 \times 10^6 \text{ mm}^2.
\]

(iii) **B.M @ Centre of Span Section**
\[
M = \frac{wL^2}{8}
\]
\[
M = \frac{1.8 \times 3^2}{8}
\]
\[
M = 14.4 \text{ kNm}.
\]

(iv) **Stress due to loads**
\[
\sigma = \frac{M}{Z}
\]
\[
\sigma = \frac{14.4 \times 10^6 \text{ Nm}}{1.05 \times 10^6 \text{ mm}^2}
\]
\[
\sigma = 13.8 \text{ N/mm}^2.
\]
(v) Prestress at top and bottom fibres,
\[
\left[ \frac{P + P_e}{A} \right] = \left[ \frac{350 \times 10^3 N + 260 \times 10^3 N \times 40 \text{mm}}{285 \times 10^3 \text{mm}^2 + 1.04 \times 10^6 \text{mm}^2} \right] = 10 \pm 9.6 \text{ N/mm}^2.
\]

(vi) Resultant stresses.
- At top: \(10 - 9.6 + 13.3 = 14.2 \text{ N/mm}^2\) [Comp]
- At bottom: \(10 + 9.6 - 13.3 = 5.8 \text{ N/mm}^2\) [Comp]

(vii) If \(P =\) prestressing force to balance the stress at soffit, then
\[
\frac{P + P_e}{A} + \frac{e}{z} = \frac{M}{z}; \text{ Here, we have}
\]
\[
P \left[ \frac{1}{A} + \frac{e}{z} \right] = \frac{M}{z}
\]
\[
P \left[ \frac{1}{285 \times 10^3} + \frac{40}{1.04 \times 10^6} \right] = \frac{14.4 \times 10^6}{1.04 \times 10^6}
\]
\[
P = 17.6 \times 10^3 \text{ N.}
\]
\[
P = 17.6 \text{ kN.}
\]
An unsymmetrical I-section beam is used to support an IL of 2 kn/m over a span of 8 m. The sectional details are top flange, 300 mm wide & 60 mm thick, bottom flange, 100 mm wide & 60 mm thick, thickness of web = 80 mm; overall depth = 400 mm. At the centre of span, the effective prestressing force of 150 kn is located at 50 mm from the back of the beam. Estimate the stress at the centre of span section of the beam for the following load conditions.

a) prestress + self wt, b) prestress + s.w + l.l [2016]

Given:
- \( p = 150 \text{ kn} \)
- \( e = 50 \text{ mm} \)
- \( L = 8 \text{ m} \)

Solution:

1. **Area:**
   
   \[
   A_1 + A_2 + A_3 = (300 \times 60) + (80 \times 280) + (100 \times 60) 
   \]
   
   \[
   A = 46400 \text{ mm}^2 
   \]

2. **Distance of centroid from bottom \( y \):**

   \[
   \bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} 
   \]

   \[
   \bar{y} = \frac{(300 \times 60) \times \left( \frac{60}{2} + 540 \right) + (80 \times 280) \times \left( \frac{280 + 60}{2} \right) + (100 \times 60) \times \left( \frac{60}{2} \right)}{46400} 
   \]

   \[
   \bar{y} = 6.6 \times 10^3 + 4.43 \times 10^3 + 130 \times 10^3 
   \]

   \[
   \bar{y} = 2439 = 24.4 \text{ mm} 
   \]

3. **My distance of centroid from top \( y \):**

   \[
   \bar{y} = 156 \text{ mm} 
   \]
(iii) 
\[ M \cdot J = I = \frac{bd^3}{12} + A_1 h_1^2 \]
\[ h = \bar{y} - y_1 \]
\[ I_{xx} = \left[ \frac{bd^3}{12} + A_1 h_1^2 \right] + \left[ \frac{bd^3}{12} + A_2 h_2^2 \right] + \left[ \frac{bd^3}{12} + A_3 h_3^2 \right] \]
\[ = \left[ \frac{300 \times 60^3}{12} + (200 \times 60) \times \left[ 244 - \left( \frac{60}{2} + 340 \right) \right]^2 \right] \]
\[ + \left[ \frac{80 \times 200^3}{12} + (80 \times 200) \times \left[ 244 - \left( \frac{60}{2} + 60 \right) \right]^2 \right] \]
\[ + \left[ \frac{100 \times 60^3}{12} + (100 \times 60) \times \left[ 244 - \left( \frac{60}{2} \right) \right]^2 \right] \]
\[ = 3.91 \times 10^6 + 139.70 \times 10^6 + 276.5 \times 10^6 \]
\[ = 759 \times 10^6 \text{ mm}^4 \]
\[ I = 759 \times 10^6 \text{ mm}^4 \]

(iv) Section modulus, 
\[ Z_L = \frac{I}{y} \]
\[ = \frac{759 \times 10^6}{156} \text{ mm}^3 \]
\[ Z_L = 4.85 \times 10^4 \text{ mm}^3 \]

\[ Z_2 = \frac{I}{y} \]
\[ = \frac{75.9 \times 10^6}{244} \text{ mm}^3 \]
\[ Z_2 = 310 \times 10^4 \text{ mm}^3 \]
(V) Self weight = \( y = \text{Area} \times \text{unit wt} \)
\[ y = 0.0464 \times 24 \]
\[ y = 1.12 \text{ kN/m} \]

Self weight moment for DL = \( M_g = \frac{W_k L^2}{8} \)
\[ M_g = \frac{1.12 \times 8^2}{8} \]
\[ M_g = 8.96 \text{ kN/m} \]

Self weight moment for LL = \( M_g = \frac{W_k L^2}{8} \)
\[ M_g = \frac{8 \times 8^2}{8} \]
\[ M_g = 16 \text{ kN/m} \]

Types of stress

- \( \sigma \) at top fibre (N/mm\(^2\))
- \( \sigma \) at bottom fibre (N/mm\(^2\))

Prestress
\[ \sigma = \frac{100 \times 10^2}{464.00} \]
\[ \sigma = 2.18 \]

\[ \sigma_e = \frac{100 \times 10^2 \times 194}{438 \times 10^4} \]
\[ \sigma_e = 4.0 \]

Self wt. stress
\[ M_g = \frac{8.96 \times 10^6}{438 \times 10^4} \]
\[ M_g = 1.97 \]

LL stress
\[ M_g = \frac{16 \times 10^6}{438 \times 10^4} \]
\[ M_g = 3.64 \]

(Vi) Resultant stresses:

a) Prestress + S. wt. stress = 5.5 N/mm\(^2\)

b) Prestress + S. wt. + LL = 12.46 N/mm\(^2\)
A prestressed concrete beam with a rectangular section 120 mm wide by 300 mm deep supports a load of 4 kN/m, which includes the self wt. of beam. The effective span is 6 m. The beam is concentrically prestressed by a cable carrying a force of 180 kN. Locate the position of pressure line in beam.

**Given:**

\[ b = 120 \text{ mm}, \quad p = 4 \text{ kN/m}, \quad L = 6 \text{ m}, \quad P = 180 \text{ kN} \]

**Solution:**

(i) Area, \( A = 120 \times 300 = 36 \times 10^3 \text{ mm}^2 \).

(ii) Section modulus, \( z = \frac{bd^2}{6} = \frac{120 \times 300^2}{6} = 18 \times 10^5 \text{ mm}^3 \).

(iii) B.M @ the centre of span = \( \frac{wL^2}{8} = \frac{4 \times 6^2}{8} = 18 \text{ kNm} \).

(iv) Direct stress = \( \frac{P}{A} = \frac{18 \times 10^5}{36 \times 10^3} = 5 \text{ N/mm}^2 \).

(v) Bending stress = \( Mz \) = \( \frac{18 \times 10^4}{18 \times 10^5} = 10 \text{ N/mm}^2 \).

(vi) Resultant stresses @ centre of span

@ Top = \( \frac{P}{A} + \frac{Mz}{z} = 5 + 10 = 15 \text{ N/mm}^2 \) (Compression).

@ Bottom = \( \frac{P}{A} - \frac{Mz}{z} = 5 - 10 = -5 \text{ N/mm}^2 \) (Tension).

(vii') If \( N = \) resultant thrust in the section

\[ e = \text{ eccentricity} \]

\[ \frac{N}{A} + \frac{Mz}{z} = 15 \]

\[ \frac{180 \times 10^3 + 180 \times 10^4 \times 2}{36 \times 10^3} = 15 \times 10^5 \]

\[ A = 180 \text{ mm} \]
Distribution of stresses at location of pressure line in prestressed beam

A PSC beam with a rectangular section 120 mm wide by 300 mm deep is used over an effective span of 6 m to support a load of 4 kN/m, which includes the s.w. of the beam. The beam is prestressed by a straight cable carrying a force of 180 kN located at an eccentricity of 50 mm. Determine the location of the thrust line and plot its position at quarter span section.

[NOV-DEC-2017]

Given:
- $b = 120$ mm, $d = 300$ mm, $l = 6$ m, $w = 4$ kN/m,
- $P = 180$ kN, $e = 50$ mm

Solution:

1) Area, $A = bd = 120 \times 300 = 36 \times 10^3$ mm$^2$.

2) Section modulus, $Z = \frac{bd^2}{6} = \frac{120 \times 300^2}{6} = 18 \times 10^2$ mm$^3$.

3) Bending moment, $M = \frac{wl^2}{8} = \frac{4 \times 6^2}{8} = 18$ kN.m.

4) Direct stress, $\sigma = \frac{P}{A} = \frac{180 \times 10^3}{36 \times 10^2} = 5$ N/mm$^2$.

5) Tensile stress, $\sigma = \frac{P \times e}{A} = \frac{180 \times 10^3 \times 50}{36 \times 10^2} = +5$ N/mm$^2$.

6) Resultant stress: $\sigma = \frac{P + \sigma}{2}$
@ top = \frac{P}{A} - \frac{P_e}{Z} + \frac{M}{Z} = 5 - 5 + 10 = 10 \text{ N/mm}^2

@ bottom = \frac{P}{A} + \frac{P_e}{Z} - \frac{M}{Z} = 5 + 5 - 10 = 0 \text{ N/mm}^2

7) Shift of pressure line = \frac{M}{P} = \frac{135 \times 10^6}{180 \times 10^3} = 75 \text{ mm}

8) B.M @ pressure span section = \frac{3WL^2}{8} = \frac{3 \times 4 \times 6^2}{8} = 13.5 \text{ kN/m}

9) Bending stress @ top and bottom = \frac{M}{z}
   = \frac{13.5 \times 10^6}{18 \times 10^5} \text{ N/mm}^2
   = 7.5 \text{ N/mm}^2

10) Resultant stress
    @ top = \frac{P}{A} - \frac{P_e}{Z} + \frac{M}{Z} = 5 + 5 + 7.5 = 7.5 \text{ N/mm}^2

    @ bottom = \frac{P}{A} + \frac{P_e}{Z} - \frac{M}{Z} = 5 + 5 - 7.5 = 2.5 \text{ N/mm}^2

11) Shift of pressure line = \frac{M}{P} = \frac{135 \times 10^6}{180 \times 10^3} = 75 \text{ mm}
Load Balancing concept.

A prestress beam supports an imposed load of 4 kN/m over an effective span of 10 m. The beam has rectangular section with a width of 200 mm, depth 600 mm. Find the effective prestressing force in the cable if it is parabolic with e = 100 mm at centre and zero at ends for the following condition.

(i) If the bending effect of prestress force nullified by imposed load for mid-span action.
(ii) If the resultant stress due to self wt. imposed load and p.e. force is zero at sofit of beam for mid-span section.

Solution:

Step 1: Properties of Section

Area of cross-section:

\[ A = 200 \times 600 = 120000 \text{ mm}^2 \]

\[ e = 100 \text{ mm} \]

\[ I = \frac{bd^3}{12} = \frac{200 \times 600^2}{12} = 3.6 \times 10^9 \text{ mm}^4 \]

\[ z = \frac{I}{y} = \frac{3.6 \times 10^9}{300} = 12 \times 10^4 \text{ mm}^3 \]

Step 2: Prestressed Force

\[ \frac{M}{Zb} - \frac{Mv}{Zb} = 0 \]

\[ \frac{M}{Zb} = \frac{Mv}{Zb} \]

\[ \frac{P_e}{Zb} = \frac{Mv}{Zb} \Rightarrow P_e = Mv \]

\[ = p \times 100 = Mv \]
\[ P \times 100 = M_q \]
\[ M_q = \frac{W L^2}{8} \]
\[ = \frac{4 \times 10^2}{8} \times 8 \times 0.1 \]
\[ P = 500 \text{ kN} \]

**Step 3:** Case (i), prestressed force.

\[ \frac{P}{A} + \frac{P e}{Z_b} = \frac{M_q}{Z_b} \]

\[ P \left[ \frac{1}{A} + \frac{1}{Z_b} \right] = \frac{M_q}{Z_b} \]

\[ P = 60000 \times 7.29 \]

\[ P = 437.50 \text{ kN} \]
UNIT II DESIGN FOR FLEXURE AND SHEAR

Basic assumptions for calculating flexural stresses.

- Plane section remains plane after bending.
- Perfect bond between concrete and prestressing steel for bonded tendons.
- The prestressed concrete section will also behave like a reinforced concrete section only after cracking.
- The tensile stresses are resisted by the steel components namely untensioned reinforcement if and the high tensile steel.

Different types of flexural failure modes observed in PSC beams

- Failure of under reinforced sections
- Failure of over reinforced sections
- Failure of sections by other modes
- Fracture of steel in tension

Strain compatibility method

The method of estimating the flexural strength of prestressed concrete sections is based on the compatibility of strains and equilibrium of forces acting on the section at the stage of failure.

The assumptions made for strain compatibility method.

- The stress distribution in the compression zone of concrete can be defined by means of coefficients applied to the characteristic compressive strength and the average compressive stress and the position of the centre of compression can be assessed.
- Plane sections normal to the axis remain plane after bending.
- The resistance of concrete in tension is neglected.
- The maximum compressive strain in concrete at failure reaches a particular value.

Factors which influence the ultimate flexural strength of PSC beams.

- The failure mainly a flexural failure, which no effect of shear, bond or anchorage, which might decrease the strength of the section.
- The beams are bonded. Unbanded beams have different ultimate strength than for bonded beams.
- Beams are statically determinate.
Ultimate loads, obtained are the result of short time static loading.
No effect of impact fatigue or long time loading is considered.

Conventional failure of an over reinforced prestressed concrete beam.
An Over reinforced members fail by the sudden crushing of concrete. The failure being reinforced members fail by the sudden crushing of concrete. The failure being characterized by small deflections and narrow cracks, the area of steel being comparatively large, the stresses developed in steel at failure of the member may not reach the tensile strength.

Type I, Type II and Type III structures.
In a type I member, no tensile stress is allowed under service loads or at transfer.
In a type II member, tensile stresses are allowed but they should be within the cracking stress.
For a type III member, the tensile stress can exceed the cracking stress, but still it is limited to a certain value which limits the crack width.

Degree of prestressing.
A measure of the magnitude of the prestressing force related to the resultant stress occurring in the structural member at working load.

Flexure failure of conventional RC beam with PSC beam.
There is no difference in flexural failure of a conventional RC beam and a prestressed concrete beam. Because the prestressed concrete beam will behave as an RC beam, after flexural cracks are formed.

Horizontal shear
The horizontal shear stress is (normally) maximum at the neutral axis of the beam. This is opposite of the behavior of the bending stress which is maximum at the other edge of the beam, and zero at the neutral axis.

Methods of improving the shear resistance of a prestressed concrete beams
- Horizontal or axial prestressing
- Prestressing by inclined or sloping cables
- Vertical or transverse prestressing

Analysis of the reinforced and prestressed concrete members under shear
The analysis for axial load and flexure are based on the following principles of mechanics
- Equilibrium of internal and external forces
- Compatibility of strains in concrete and steel
- Constitutive relationship of materials.

Different types of cracks in a simply supported beam under uniformly distributed load without prestressing.
Flexural cracks – These cracks form at the bottom near the midspan and propagate upwards.

Web shear cracks – These cracks form near the neutral axis close to the support and propagate inclined to the beam axis.

Flexure shear cracks – These cracks form at the bottom due to flexure and propagate due to both flexure and shear.

**Functions of stirrups.**
- Stirrups resist part of the applied shear.
- They restrict the growth of diagonal cracks.
- The stirrups counteract widening of the diagonal cracks, thus maintaining aggregate interlock to a certain extent.
- The splitting of concrete cover is restrained by the stirrups, by reducing dowel forces in the longitudinal bars.

**Unbounded Tendons**

For members with unbounded tendons and with the span/depth ratio not exceeding 35, the stress in the tendons is computed by the relation,

$$F_{ps} = (f_{ps} + 70 + (f_{c}/100)p)$$

**Factors to be considered in design of prestressed concrete section for flexure.**

Two stages of loading are to be considered in design of prestressed concrete section for flexure are as, 1) Transfer of prestressing force, 2) At working load (service stage)

**principles of mechanics for the analysis of axial load and flexure in PSC structures.**

The analysis for axial load and flexure are based on the following principles of mechanics.
- Equilibrium of internal and external forces.
- Compatibility of strains in concrete and steel.
- Constitutive relationships of materials.

**Factors which influence the ultimate flexural strength of PSC beams.**

- The failure mainly a flexural failure, with no effect of shear, bond or anchorage which might decrease the strength of the section.
- The beams are bonded. Unbonded beams have different ultimate strength than for bonded beams.
- Beams are statically determinate.
- Ultimate loads, obtained is the result of short time loading. No effect of impact fatigue or long time loading is considered.
LAYOUT OF CABLES IN POST TENSION BEAMS
LAYOUT OF CABLES IN PRE TENSION BEAMS
Design of Shear Reinforcement:

At any section, the ultimate shear resistance Vc of the concrete alone should be taken as the lesser of the values of Vcw and Vcg, when V, the shear force due to the ultimate loads is less than Vc, the shear force which can be carried by the concrete, minimum shear reinforcement should be provided in the form of stirrups such that,

\[ S_v = \left( \frac{Axv \cdot 0.875f_y}{0.45d} \right) \]

where,

- \( S_v \): Spacing of stirrups along the length of member
- \( Axv \): Total cross-sectional area of stirrups less effective in shear
- \( b \): breadth of the member (in m)
- \( f_y \): characteristic strength of the stirrup reinforcement (in MPa)

If the shear force V is less than 0.5Vc and in a member of minor importance, shear reinforcement need not be provided.

When V exceeds Vc, shear reinforcement is required conforming to the relation,

\[ S_v = Axv \cdot 0.875f_y d_t \]

\[ \frac{V}{V_c} \]

Where,

- \( d_t \): depth from the extreme compression fibre

The spacing of stirrups should exceed neither 0.75d_t nor four times the clear thickness for flanged members. When V exceeds 1.2Vc, the maximum spacing should be reduced to 0.5d_t.

The lateral spacing of the individual legs of the stirrups provided at a cross-section should not exceed 0.75d_t. If the nominal shear stress \( \left( \frac{V}{d_t} \right) \) exceeds the maximum shear stress values, the section has to be redesigned.
Design of Flexure:

Design a simply supported type I prestressed beam with $M_t = 435 \text{ kN} \cdot \text{m}$ (including an estimated $M_{sw} = 55 \text{ kN} \cdot \text{m}$). The height of the beam is restricted to 920 mm. The prestress at transfer $f_p = 1035 \text{ N/mm}^2$ and the prestress at service $f_{pc} = 860 \text{ N/mm}^2$. Based on the grade of concrete, the allowable compressive stresses are $12.5 \text{ N/mm}^2$ at transfer and $11 \text{ N/mm}^2$ at service.

The properties of the prestressing strand are given below.

Type of prestressing tendon = 7 wire strand

Nominal diameter = 12.7 mm

Nominal area = 99.3 mm$^2$.

Solution: A) Preliminary design:

Given, $h = 920 \text{ mm}$, $f_p = 1035 \text{ N/mm}^2$, $M_t = 435 \text{ kN} \cdot \text{m}$

1) Estimate the lever arm ($x$)

$$D \cdot 3.27 = 130.5 > 55 \text{ kN} \cdot \text{m}$$

$$x = 0.5 \times h = 460 \text{ mm}.$$ 

2) Estimate the effective prestress ($f_{pc}$) since $M_{sw}$ is small.

$$f_{pc} = \frac{M_{IL}}{Z}$$

$$= \frac{(435 - 55) \times 10^6}{460}$$

$$= 326 \text{ kN}.$$ 

3) Area of prestressing steel ($A_p$)

$$A_p = \frac{f_{pc} \cdot 10^2}{860}$$

$$= 960 \text{ mm}^2.$$
1. Estimate the area of the section to have average stress in concrete equal to 0.5 tfc, at

\[ A = \frac{P_e}{0.5 \text{ tfc}} \]

\[ = \frac{32.6 \times 10^3}{(0.5 \times 112)} \]

\[ = 150 \times 10^3 \text{ mm}^2. \]

The following trial section has the required depth and area.

Total cross-section

4. Geometric properties:

(i) \( Ge = \frac{920}{2} = 460 \text{ mm} \)

(ii) Area = \( (450 \times 100) \times 2 + (720 \times 100) + \frac{450 \times 220}{12} \)

\[ = 142 \times 10^3 \text{ mm}^2. \]

(iii) \[ I = 2 \left[ \frac{12}{12} \times 450 \times 100^2 + (450 \times 100) \times (410^2) \right] + \frac{1}{12} [100 \times (720^2)]. \]

\[ = 1.33 \times 10^8 \text{ mm}^4. \]

(iv) \[ y^2 = \frac{y}{A} = \frac{112.76 \times 10^3}{12} \text{ mm}^2. \]

(v) Beam levels

\[ k_b = k_L = y^2 = 245.5 \text{ mm.} \]

C. Final Design.

4. Calculate the eccentricity, \( e \):

\[ P_0 = A_p + P_o \]

\[ = 960 \times 10^3 = 993.6 \text{ kN}. \]

\[ e = \frac{P_m h + k_L}{P_0} \]

\[ = \frac{55 \times 10^6 + 245.5}{993.6 \times 10^3} \]

\[ = 300.8 \text{ mm}. \]
2. Recompute the effective prestress and associated variables,

\[ P_e = \frac{M_t}{E + k_t} = \frac{486 \times 10^6}{580 + 245.5} = 777 \text{ kN} \]

Since, \( P_e \) obtained is less than the previous value, the properties of prestress are recomputed,

\[ A_p = \frac{P_e}{\sigma_p} = \frac{777 \times 10^5}{860} = 926.7 \text{ mm}^2 \]

\[ P_0 = A_p \times d_{p_e} = 926.7 \times 1.035 \]

\[ e = \frac{M_{ew}}{P_0} = 5.02 \text{ mm} \]

From the above properties,

\[ P_e = \frac{M_t}{E + k_t} = \frac{486 \times 10^6}{580 \times 245.5} = 740.5 \text{ kN} \]

Which is closer to the estimated value of \( P_e \) (777 kN).

The tendons are placed in 2 ducts.

Select 10-7-wire strands with

\[ A_p = 10 \times 97.3 \]

\[ = 973 \text{ mm}^2 \]

3) Check the compressive stress in concrete

a) At transfer,

\[ A \geq \frac{P_0 h}{t_{con} \cdot 0.8 \times 1.4} = \frac{959 \times 10^2 \times 920}{11 \times 460} = 152.4 \text{ mm}^2 \]

b) At service

\[ A \geq \frac{P_0 h}{t_{con} \cdot 1.4} = \frac{754.5 \times 10^3 \times 920}{11 \times 460} = 144.4 \text{ mm}^2 \]
For Question No. 1, Type 2 Design

A) Preliminary design:

Given,

\[ h = 920 \text{ mm}, \quad M_T = 435 \text{ kNm}, \quad M_{sw} = 55 \text{ kNm} \]

Since \( h, M_T, M_{sw} \) are already given, steps 1 to 5 can be omitted.

6) Estimate the lever arm (\( Z \))

\[
0.3 \times M_T = 0.3 \times 435 = 130.5
\]

\[ Z = 0.5h = 0.5 \times 920 = 460 \text{ mm}. \]

7) Estimate the effective prestress

\[
P_e = \frac{M_{sl}}{Z}
\]

\[ M_{sl} = M_T - M_{sw} = 435 - 55 = 380 \text{ kNm} \]

\[ P_e = \frac{380 \times 10^6}{460} = 826 \text{ kN}. \]

8) Estimate the area of prestressing steel

\[
A_p = \frac{P_e}{d_{pe}}
\]

\[ A_p = \frac{826 \times 10^3}{460} = 1.8 \text{ mm}^2 \]

9) Estimate the area of the section to have average shear in concrete equal to 0.5 kN/\( \text{mm} \)

\[
A = \frac{P_e}{0.5 \text{ kN/}\text{mm}} = \frac{826 \times 10^3}{0.5 \times 11} = 150 \times 10^3 \text{ mm}^2
\]

The following steel section has the required depth and area.
B) Geometric properties.

\[ A = 2 \left( 370 \times 100 + 720 \times 100 \right) \]
\[ = 150000 \text{ mm}^2 \]

\[ I = 2 \times \left[ \frac{1}{12} \times 370 \times 100^3 + (370 \times 100 \times 410)^2 \right] \]
\[ = 1.6287 \times 10^{10} \text{ mm}^4 \]

\[ \phi^2 = \frac{I}{A} = \frac{1.6287 \times 10^{10}}{150 \times 10^3} = 108530 \text{ mm}^2 \]

\[ k_L = k_B = \frac{\phi^2}{c_t} = \frac{108530}{460} = 236 \text{ mm} \]

Summary of preliminary design:

\[ A = 150 \times 10^3 \text{ mm}^2 \]

\[ I = 1.6287 \times 10^{10} \text{ mm}^4 \]

\[ c_t = c_B = 460 \text{ mm} \]

\[ k_L = k_B = 236 \text{ mm} \]

\[ A_p = 960 \text{ mm}^2 \]

\[ P_e = 826 \text{ KN} \]

C) Final Design.

1. Calculate the eccentricity, \( e \):

\[ e_1 + e_2 = \frac{M_{sw} + f_{cu} t_k c_k A_k b}{P_0} \]

\[ = \frac{55 \times 10^6 + (2.1 \times 150 \times 10^{-3} \times 230)}{793.6 \times 10^3} \]

\[ = 120.3 = 120 \text{ mm} \]

\[ e = e_1 + e_2 + k_B b \]

\[ = 130 + 23.6 \]

\[ = 366 \text{ mm} \]

\[
pe = \frac{M_t - \frac{1}{2}L_v \Delta eL}{\frac{e + \frac{1}{2}L_v}{L_v}}
\]

\[
= \frac{435 \times 10^4 - 1.65 \times 10^5 \times 10^2 \times 2.36}{2.36 + 2.36}
\]

\[
= 625.6 \text{ kN.}
\]

Since $pe$ is substantially lower than the previous estimate of 326 kN, $Ap$, $po$ and $e$ need to be recalculated.

\[
Ap = \frac{pe}{\frac{1}{2}pe}
\]

\[
= \frac{625.6 \times 10^3}{760} = 727 \text{ mm}^2
\]

3. Recompute eccentricity ($e$).

\[
p_0 = Ap \times \frac{1}{2}po
\]

\[
= 727 \times 10^3
\]

\[
= 752.4 \text{ kN.}
\]

\[
e = \frac{M_{sw} + \frac{1}{2}L_v \Delta eL, cL_v A_{cb} + k_b}{p_0}
\]

\[
= \frac{55 \times 10^6 + 2.1 \times 150 \times 10^2 \times 2.36 + 2.36}{752.4 \times 10^3}
\]

\[
= 403 \text{ mm} = 400 \text{ mm.}
\]

4. Check the compressive stress in concrete.

a) At transfer

\[
A \geq \frac{p_0 L}{f_{cc}, L \Delta e, aL \Delta e, cL} = 138352 \text{ mm}^2.
\]

b) \[
A \geq \frac{pe L}{f_{cc}, cL \Delta e, aL \Delta e, cL} = 126631 \text{ mm}^2.
\]
A prestressed prestressed beam of rectangular section 350 mm wide is to be designed for an imposed load of 1.2 kN/m uniformly distributed on a span of 12 m. The stress in the concrete must not exceed 17 N/mm² in compression or 14 N/mm² in tension at any time and the loss of prestress may be assumed to be 15 percent.

Calculate

1) the minimum possible depth of the beam and
2) for the section provided, the minimum prestressing force and the corresponding eccentricity.

Solution:

\[ q = 1.2 \text{ kN/m} \quad h = 0.5 \quad b = 350 \text{ mm} \quad t_{tw} = 17 \text{ kN/mm}^2 \]

Overall depth of section = \( h \text{ mm} \).

\[ h = t_{tw} = 17 \text{ mm} \]

Live load moment \( M_L = \frac{L^2}{2} \) = 216 kN.m.

Dead load moment \( M_d = \frac{bh^2}{12} \times \frac{1}{12} \) = 216 \( \text{kN.m} \)

Range of stress at bottom fibre \( \sigma_{br} = (h + t_{tw} - (\eta/2)) \)

\[ = [0.5 + 17 - (-14)] \]

\[ = 15.95 \text{ N/mm}^2 \]

a) Minimum section modulus,

\[ Z_b = \left[ \frac{M_g + (1-h)M_g}{T_{by}} \right] \]

\[ \frac{bh^2}{6} = \left[ \frac{216 \times 10^6 + (1-0.95)432bh}{15.95} \right] \]

10.35bh² = 1.276 \times 10^6 + 2.592bh - 2.2bh

15.95bh² = (216 \times 10^6 + 432bh - 364 \times 2bh)

\[ h = 580 \text{ mm} \].
b) For the section provided, \( b = 250 \text{mm}, h = 580 \text{mm} \):

\[
A = 14 \times 10^3 \text{ mm}^2.
\]

\[
Z_b = Z_L = 14 \times 10^6 \text{ mm}^3.
\]

\[
M_g = 625 \times 10^5 \text{ Nmm}.
\]

\[
(M_g + M_q) = 2735 \times 10^5 \text{ Nmm}.
\]

\[
\tau_{sup} = \left( \frac{t_k}{t_L} - \frac{M_g}{Z_L} \right)
= \left( -1.4 - \frac{652 \times 10^5}{14 \times 10^6} \right) = -5.9 \text{ N/mm}^2.
\]

\[
\tau_{int} = \left[ \frac{t_L}{Z_b} + \frac{(M_g + M_q)}{Z_b} \right]
= \left[ \frac{-1.4 + 2735 \times 10^5}{0.35 \times 14 \times 10^4} \right]
= -22 \text{ N/mm}^2.
\]

Minimum prestressing force:

\[
p = \left[ A \left( \frac{\tau_{int} Z_b + \tau_{sup} Z_L}{Z_b + Z_L} \right) \right]
= 1170 \text{ kN}.
\]

Corresponding eccentricity \( u \):

\[
e = \left[ \frac{Z_b Z_L (\tau_{int} - \tau_{sup})}{A \left( \frac{\tau_{sup} Z_L + \tau_{int} Z_b}{Z_b + Z_L} \right)} \right]
= 167.5 \text{ mm}.
\]
Design of section for the limit state of collapse in flexure.

A pretensioned prestressed concrete beam of rectangular section is required to support a design ultimate moment of 150 kNm. Design the section, if fck is 50 N/mm² and fp = 1600 N/mm².

Solution:

\[ \frac{b}{d} \rightarrow \text{breadth and effective depth of the section,} \]

\[ \frac{2u}{d} = 0.5 \]

Assume \( \frac{2u}{d} = 0.5 \)

\[ Mu = 0.14 fck b d^2 \]

\[ Mu = 0.86 fck b \times (d - 0.42xu) \times (d - 0.42xu) \times (d - 0.42xu) \times (d - 0.42xu) \times (d - 0.42xu) \]

Assume d = 0.5d

\[ Mu = 0.14 fck (0.5d) d^2 \]

Simplifying:

\[ d^3 = \left( \frac{Mu}{0.14 fck \times 0.5} \right) \]

\[ = \left( \frac{100 \times 10^6}{0.14 \times 50 \times 0.5} \right) \]

\[ d = 300 \text{ mm}, \quad b = 150 \text{ mm} \]

\[ \frac{xu}{d} = 0.5 \]

form Table 7.1 (Table 11 of IS 1342:1980)

\[ fp = 0.87 f_p \]

\[ \therefore \quad Ap = \frac{Mu}{0.87 f_p (d - 0.42xu)} \]

\[ = \left( \frac{100 \times 10^6}{0.87 \times 1600 \times (300 - 0.42 \times 150)} \right) \]

\[ = 300 \text{ mm}^2 \]

Adopt a section, 150 mm wide by 250 mm deep with 500 mm² of high tensile steel located at an effective depth of 500 mm.
A pretensioned T-section has a flange 1200 mm² wide and 150 mm thick. The width and depth of the rib are 300 and 1500 mm respectively. The high tensile steel has an area of 470 mm² and is located at an effective depth of 700 mm. If the characteristic cube strength of the concrete and the tensile strength of steel are 40 N/mm² and 1600 N/mm² respectively, calculate the flexural strength of the T-section.

Solution:

\[ A_P = 470 \text{ mm}^2 \quad d = 1600 \text{ mm} \quad f_p = 1600 \text{ N/mm}^2 \]
\[ f_{ck} = 40 \text{ N/mm}^2 \quad D_f = 1600 \text{ mm} \quad b = 1200 \text{ mm} \]
\[ b_w = 300 \text{ mm} \quad A_{pw} = A_{pw} + A_{pf} \]

\[ A_{pf} = 0.45 f_{ck} (d - b_w) \left( \frac{D_f}{d} \right) \]
\[ = (0.45 \times 40) (1200 - 300) \left( \frac{1600}{1600} \right) \]
\[ = 151.2 \text{ mm}^2. \]
\[ A_{pw} = 4700 - 151.2 \]
\[ = 3182 \text{ mm}^2. \]

Also
\[ \frac{A_{pw}}{d_w d_f f_{ck}} = \frac{3182 \times 1600}{300 \times 1600 \times 40} = 0.265 \]

\[ + \frac{f_{pu}}{0.87 f_p} = 1.0 \]

\[ f_{pu} = 0.87 \times 1600 = 1372 \text{ mm}^2. \]
\[ \frac{x_u}{d} = 0.56 \]
\[ x_u = 0.56 \times 1600 = 896 \text{ mm}. \]

\[ M_u = f_{pu} A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (d - b_w) D_f \left( d - 0.5 D_f \right) \]
\[ = (1372 \times 3182) (1600 - 0.42 \times 896) + 0.45 \times 40 \times (900) \times \]
\[ = \left[ (18420 \times 10^6) + (3705 \times 10^6) \right] \times \frac{150}{1600 - 75} \]
\[ = 91.25 \times 10^6 \text{ Nmm}. \]
\[ = 91.25 \text{ KNmm}. \]
UNIT III DEFLECTION AND DESIGN OF ANCHORAGE ZONE

Factors affecting the deflection of the prestressed concrete beam
- Imposed load & self weight
- Magnitude of the prestressing force.
- Second moments of area of cross section
- Shrinkage, creep & relaxation of steel stress.
- Modulus of elasticity of concrete
- Cable profile
- Span of the member
- Rigidity condition

Importance of control of deflection.
1. Excessive sagging of principal structural members is not only unsightly, but at times, also renders the floor unsuitable for intended use.
2. Large deflections under dynamic effects and under the influence of variable loads may cause discomfort to the users.
3. Excessive deflections are likely to cause damage to finishes, partitions and associated structures.

Mohr’s theorem
- Short term or instantaneous deflection of prestressed members are governed by the bending moment distribution along the span and the flexural rigidity of the members.
- Mohr’s moment area theorem are readily applicable for the estimation of deflections due to the prestressing force, self weight and imposed loads.

Then, according to mohr’s first theorem,
Slope = area of BMD/ flexural rigidity

\[ \theta = \frac{A}{EI} \]

Mohr’s second theorem states that,

Intercept, \( \Delta \) = Moment of the area at BMD

\[ \Delta = \frac{Ax}{EI} \]

The deflection of symmetrically loaded and simply supported beams at the mid-span point are directly obtained from the second moment area theorem since the tangent is horizontal at this point. More complicated problems involving unsymmetrical loading may be solved by combining both the moment area theorems.
The beam AB is subjected to a bending moment distribution due to the prestressing force or self-weight or imposed loads. ACB is the central line of the deformed structure under the system of given loads.

If \( \theta \) - Slope of the elastic curve at A

AD - intercept between the tangent at C and the vertical at A

\( \delta \) - deflection at the centre for symmetrically loaded,

Simply supported beam

A - area of the bending moment diagram between A and C

x - distance of the centroid of the BMD between A and C from the left support.

EI - flexural rigidity of the beam
Anchorage zone.

Prestressed concrete contains tendons which are typically stressed to about 1000 MPa. These tendons need to be anchored at their ends in order to transfer (compressive) force to the concrete. The zone of region is called Anchorage zone.

Functions of end blocks.

- Provide Lateral (horizontal) stability from wind and other horizontal (Racking) loads.
- Provide additional vertical load capacity for the ends of the joists from point loads above.

stress distribution in end block.(anchorage zone)

A physical concept of the state of stress in the transverse direction, that is normal to planes parallel with the top and bottom surfaces of the beam, may be obtained by considering these lines of force as individual fibres acting as curved stuts between end force 2P and the main body of the beam.

By providing an external initial stress (the prestress) which compresses the beam. Now they can only separate if the tensile stress induced by the self weight of the beam is greater than the compressive prestress introduced.

Effective reinforcement ratio

Ratio of effective area of reinforcement to the effective area of concrete at any section of a structural member is known as effective reinforcement ratio.
End block in a post tensioned member.

The zone between the end of the beam and the section where only longitudinal stress exists is generally referred to as the anchorage zone or end block.

Bursting tension.

The effect of transverse tensile stress is to develop a zone of bursting tension in a direction perpendicular to the anchorage force resulting in horizontal cracking.

Permissible limits for deflection

The final deflection due to all loads including the effects of temperature, creep & shrinkage should not normally exceed span / 250.

Deflections including the effects of temperature, creep & span / 350 or 20mm whichever is less

If finishes are to be applied to prestressed concrete members the total upward deflections should not exceed span.

Check for serviceability limit state of deflection

It is the general practice in most of the codes to safeguard against excessive deflections under serviceability limit states, either indirectly by prescribing a minimum span to depth ratio for the member, or directly by specifying a maximum permissible deflection expressed as a fraction of the span.

The recommendations of the Indian standard code (IS 1343) with regard to the limit state of deflection are as follows,

1. The final deflection, due to all loads including the effects of temperature, creep and shrinkage should normally not exceed span / 250.

2. The deflection, including the effects of temperature, creep and shrinkage occurring after the erection of partitions and the application of finishes, should not normally exceed span / 350 or 20mm, whichever is less.

3. If finishes are to be applied to the prestressed concrete member, the total upward deflection should not exceed span / 300, unless uniformly of camber between adjacent units can be ensured.

Primary moment: The primary moment is the apparent bending moment at a section in a statically indeterminate structure due to the ahead eccentricity of the tendons from the additional moments.

Secondary moment: Secondary moments are additional moments induced at a section due to the redundant reactions developed as a consequence of prestressing the structure.
Resultant moment: The resultant moment at a section of an indeterminate prestressed structure is the sum of primary & secondary moments (i.e) \( R.M = (P.M + S.M) \)

Importance of providing reinforcement in anchorage zone or end block
In post tensioned members, the prestressing wires are introduced in cables holes or ducts. Pre-formed in the members and then stressed and anchored at the end forces. Large forces concentrated over relatively small areas are applied on the end blocks. It is linearly distributed and develop transverse and shear stressed so that an adequate amount of steel is properly distributed to sustain the transverse tensile stressed.

Transmission length
In a pretensioned system, when a wire is released, the transmission of prestressing force from steel to concrete is though a bond comprising of adhesion, friction and shearing resistance. When the bond stress is zero, and uniform stress distribution is prevalent from the section then the length for achieving this is termed as transmission length.

Necessity of using anchor block in prestressed concrete
Anchor block having higher discontinuous force applied at the end develop transverse and shear stresses. The distribution of stressed in the anchorage zone, can provide an adequate amount of steel, properly distributed to sustain the transverse tensile stresses.

Anchorage zone
Large amount of prestressing forces, concentrated over relatively small areas are applied on the end blocks through bearing plates. These forces develop transverse and shear stressed. Generally bursting tensile forces or splitting tensile force and spalling tensile forces are developed while transmitting the prestress to the concrete. These forces are resisted by providing suitable arrangements of reinforcement in the end blocks.

Investigations on anchorage zone stresses.
The stress distribution in the anchorage zone done by,
1. Magnel’s method
2. Guyon’s method
3. Zielinski and Rowe’s method.
4. Codal provision
   - Indian code
   - British code
   - American code.
In this method, the cross-section is divided into a deep beam subjected to anchorage concentrated loads on one side and the linear direct stress distribution and the parabolic shear stress distribution across the section, on the other side. The bending moment $M$, the direct vertical force $H$, and the horizontal shearing force $V$ are calculated on any required horizontal axis parallel to the soffit of the beam, usually on an axis at which horizontal shearing force is greatest.

Vertical stress, $f_v = k_1 \left( \frac{M}{BD} \right) + k_2 \left( \frac{H}{BD} \right)$

Shear stress, $f_{sh} = k_3 \left( \frac{V}{BD} \right)$

Direct stress, $f_h = \frac{P}{A} + \frac{P_0}{x}$

Finally, the principal stresses are computed by the general equations:

$$f_{max} = \frac{f_v + f_h}{2} + \frac{1}{2} \sqrt{(f_h - f_v)^2 + 4f_{sh}^2}$$

$$f_{min} = \frac{f_v + f_h}{2} - \frac{1}{2} \sqrt{(f_h - f_v)^2 + 4f_{sh}^2}$$

$$\tan \theta = \frac{2f_{sh}}{f_v - f_h}$$

Where, $k_1, k_2, k_3$ - Constants.
3) Guyon's Method:

In Guyon's method, design tables are used for the computation of bursting tension in the end blocks. The tables are based on his mathematical study of stress distribution in end blocks due to concentrated loads acting on it. The concept of symmetrical or equivalent prism for eccentric cables and the method of positioning for the analysis of sheaves developed due to multiple cables have been introduced by Guyon.

There are two distinguished cases of force distribution.

They are,
- Forces evenly distributed
- Forces not evenly distributed

3) Indian Code Provision:

For the calculation of the permissible bearing stresses and the bursting tensile force, specific guidelines are given in the IS 1843. Method for finding the actual bearing stresses and the size of the bearing is not given. The details are given below:

\[
\text{Bearing Stress} = 0.48 \times \sqrt{\frac{Abx}{Apun}} \text{ or } 0.876\]

\[
\text{where} \quad \frac{Abx}{Apun} = \frac{2500}{A_{p}} \]

\[
\text{and} \quad A_p = \frac{22}{6} \times \frac{A_{p}}{A_{n}} \]

\[
\frac{A_{p}}{A_{n}} = \frac{1}{2} \left(1 + \frac{1}{C}ight) \]

\[
\text{for} \quad C < 20 \quad \text{or} \quad C > 20
\]

\[
\frac{A_{p}}{A_{n}} = 0.5 \quad \text{for} \quad C = 20
\]
where,

$f_{ci}$ - the cube strength at transfer

$A_{bi}$ - the maximum area of that portion of the member which is geometrically similar and concentric to the effective punching area.

$A_{pi}$ - effective punching area shall generally be the contact area of the anchorage devices which, if circular in shape, shall be replaced by a square of equivalent area.

Bursting tension force

$F_{bt} = P_k \left[ 0.3 - 0.3 \left( \frac{Y_{po}}{Y_o} \right) \right]$

where,

$F_{bt}$ - bursting tension force

$P_k$ - load in the tendon assumed above

$Y_{po}$ - side of loosed area

$Y_o$ - side of end block.
16 marks:

A prestressed concrete beam (span = 10 m) of rectangular section, 120 mm wide and 300 mm deep, is axially prestressed by a cable carrying an effective force of 150 kN. The beam supports a total uniformly distributed load of 5 kN/m which includes the self wt of member. Compare the includes the self wt of member. Compare the magnitude of principal tension developed in the beam with and without axial prestress.

Solution:

Step 1: properties of section:

\[ A = 120 \times 300 = 36 \times 10^3 \text{ mm}^2 \]

\[ I = \frac{bd^3}{12} = \frac{120 \times 300^3}{12} = 27 \times 10^7 \text{ mm}^4 \]

\[ W_d = 5 \text{ kN/m} \]

Shear force at support, \[ V = \frac{W_d}{2} = \frac{5 \times 10}{2} = 25 \text{ kN} \]

Max. shear stress at support (\[ T_v \]) = \[ \frac{V}{6A} \]

\[ T_v = \frac{25 \times 10^3}{2 \times 120 \times 300} = 1.05 \text{ N/mm}^2 \]
Step 2:

Case (i): principal stresses

\[ f_x = f_y = 0 \]

\[ \frac{f_x + f_y + \sqrt{f_x^2 - 4f_y^2}}{2} + 4\times 10^2 \]

\[ = \frac{1}{2} \sqrt{4 \times 10^2} \]

\[ = \pm 1,050 \text{ N/mm}^2 \]

Step 3:

Case (ii): with axial prestress (P)

\[ f_x = \frac{130 \times 10^2}{36 \times 10^2(A)} = 5 \text{ N/mm}^2 \]

maximum & minimum principal stress

\[ = \frac{f_x}{2} \pm \frac{1}{2} \sqrt{f_x^2 + 4\times 10^2} \]

\[ = \frac{5}{2} \pm \frac{1}{2} \sqrt{5^2 + 4 \times 10^2} \]

\[ = 2.5 \pm 2.73 \]

\[ = 5.23 \text{ N/mm}^2 \]

\[ = -0.23 \text{ N/mm}^2 \]

Hence, with axial prestress, the principal tension is reduced by,

\[ \left[ \frac{1.05 - 0.23}{1.05} \right] \times 100 = 78\% \]

Step 4:

When cable is curved @ e = 150 mm.

Slope of cable at support = \[ \frac{4e}{L} = \frac{4 \times 100}{10 \times 1000} \]

\[ = 0.04 \text{ radian} \]

Vertical component of P = \[ P \sin \theta \]

\[ = 180 \times 0.04 \]

\[ = 7.2 \text{ kN} \]
Horizontal component of prestress force \( P \text{ cos } \alpha \)  
\[ = 1 \times 130 = 130 \text{ kN} \]

Net shear at support \( V = 25 - 7.2 \)  
\[ = 17.8 \text{ kN} \]

Maximum shear stress \( \tau \)  
\[ = \frac{3}{2} \cdot \frac{V}{bh} \]
\[ = \frac{3}{2} \cdot \left[ \frac{17.8 \times 10^3}{130 \times 300} \right] \]
\[ = 0.74 \text{ N/mm}^2 \]

\[ f_2 = \frac{180 \times 10^{-3}}{130 \times 300} = 6 \text{ N/mm}^2 \]

\[ f_{\text{max}} = \frac{5}{2} \pm \frac{1}{2} \sqrt{5^2 + 4 \times 0.74^2} \]
\[ = 2.5 \pm 2.62 \]
\[ = 5.12 \text{ N/mm}^2 \text{ (compression)} \]
\[ = -0.12 \text{ N/mm}^2 \text{ (tension)} \]

Reduction in principal tensile stress is
\[ \left[ \frac{0.23 - 0.12}{0.23} \right] \times 100 = 43\% \]

Reduction in principal tension
\[ \left[ \frac{1.05 - 0.12}{1.05} \right] \times 100 = 88.5\% \]
A concrete beam having a rectangular section, 150 mm wide and 300 mm deep, is prestressed by a parabolic cable having an eccentricity of 100 mm at centre of span, reducing to zero at the supports. The span of beam is 8 m. The beam supports a live load of 2 kn/m. Determine the effective force in the cable to balance the dead and live loads on beam. Estimate the principal stresses at the support section.

Solution,

Step 1: To find prestressing force:

Self wt. of beam = 0.15 × 0.30 × 24 = 1.08 kn/m.

Total load (W) = 1.08 + 2 = 3.08 kn/m

\[ M = W \frac{a^2}{8} \]

\[ P \times a = \frac{3.08 \times 8000^2}{8} \]

\[ P = 246.4 \text{ kN} \]

Slope of cable at support \( \theta = \frac{4a}{L} = \frac{4 \times 150}{8 \times 1500} = 0.2 \)

Step 2: To find principal stress

Vertical component of prestressing force = \( p \sin \theta = p \cos \theta \)

\[ = 246.4 \times 0.2 \]

\[ = 12.32 \text{ kN} \]

Reaction at support due to dead and live load = \( W \frac{a}{2} \)

\[ = 3.08 \times 8 \frac{a}{2} \]

\[ = 12.32 \]

Hence, net shear force \( V \) at support = 12.32 - 12.32

\[ V = 0 \]

Horizontal prestress at support = \( \frac{P}{A} \)

\[ = \frac{246.4}{150 \times 300} \]

\[ f_a = 8.5 \text{ N/mm}^2 \]

\[ \pm \text{ principal stress} \]

\[ = \frac{tx + dy + \sqrt{(tx - dy)^2 + z^2}}{2} \]

\[ = \frac{8.5 + (8.5)^2}{2} = 5.5 \text{ N/mm}^2 \text{ (compression).} \]
Based on Transmission length

Calculate the transmission length at end of a pre-tensioned beam as per Hoyer method using following data:

Span of Beam = 50m
Diameter of wires used = 7 mm

Coefficient of friction between steel and concrete $\mu_s = 0.30$, $\mu_c = 0.15$, $E_s = 210$ kN/mm², $E_c = 30$ kN/mm²

$\sigma_{mu} = 1600$ N/mm², $\sigma_{pi} = 0.7$, $\sigma_{pe} = 1050$ N/mm².

$\sigma_{pi} = 0.6$, $\sigma_{mu} = 900$ N/mm².

Solution:

Using Hoyer’s expression

$$L_t = \frac{\phi}{2\mu} \left[ \frac{\sigma_{pi} + \sigma_{pe}}{\sigma_{mu}} \right] \left[ \frac{\sigma_{pe}}{2 + \sigma_{pe}} \right]$$

$$L_t = \frac{\phi}{2 \times 0.1} \left[ \frac{0.7 + 1500}{0.30 - 0.7 \times 1500} \right] \left[ \frac{900}{2 \times 1050 - 900} \right]$$

$$L_t = 100 \text{ ft}$$

$$L_t = 700 \text{ mm}.$$

Note: If the beam is simply supported over a span of 50m, there should be at least 700 mm of beam projection beyond the centre of supports at each end.

**Overall length** = \((50 + 2 \times 0.7)\)

\= \(51.4\) mm.
Based on Anchorage Zone.

The end block of a post-tensioned prestressed concrete beam, 300mm wide and 300mm deep, is subjected to a concentric anchorage force of 832800N by a freemint anchorage of area 11720mm². Design and detail the anchorage reinforcement at end block.

Given: Prestressing force, \( P = 832800 \) N
Area = 300mm x 300mm, \( A = 11720 \) mm²

Solution: Step 1: Anchorage diameter,

\[
\phi = \frac{4}{\pi} \times \phi_0^2
\]

\[
2\phi_0 = \sqrt{A \times 4} = \sqrt{11720 \times 4} \approx 123 \text{ mm}
\]

\[
\frac{\phi_0}{\phi} = \frac{123}{300} = 0.41
\]

Step 2: To find bursting tension

As per IS code, \( I_{bt} = P \left[ 0.32 - 0.3 \frac{\phi_0}{\phi} \right] \)

\[
= 832800 \left[ 0.32 - 0.3 \times 0.41 \right] = 166.56 \text{ kN}
\]

Assuming 8mm diameter bars.

\[
A_t = \frac{166.56 \times 10^3}{0.37 \times 260} = 736.33 \text{ mm}^2
\]

No. of bars = \( \frac{736.33}{8} \approx 92 \) bars.

The reinforcement is to be arranged in the zone between 0.1\( \phi_0 \) to \( \phi_0 \) = 30 mm to 300 mm.
A pretensioned beam of rectangular section 250 mm wide and 750 mm deep is stressed by 1600 mm$^2$ of high-tensile steel located at an effective depth of 600 mm. The effective stress in the tendons after all losses is 800 N/mm$^2$. The beam is reinforced with supplementary reinforcement consisting of 4 bars of 25 mm diameter of Fe-450 grade steel located 100 mm from top.

Estimate the ultimate flexural strength of the section according to British code regulations. Assume the characteristic cube strength of concrete as 40 N/mm$^2$.

Given data:

\[
\begin{align*}
A_{ps} &= 1600 \text{ mm}^2 \\
A_s &= 1964 \text{ N/mm}^2 \\
\sigma_p &= 1600 \text{ N/mm}^2 \\
\sigma_e &= 800 \text{ N/mm}^2 \\
\sigma_{cu} &= 40 \text{ N/mm}^2 \\
\end{align*}
\]

The pretensioned reinforcement is replaced by an equivalent area of post-tensioning tendons, which is given by

\[
\left( \frac{A_s + \sigma_y}{\sigma_p} \right) = \left( \frac{1964 \times 450}{1600} \right) = 564 \text{ mm}^2.
\]

\[
\therefore \text{ Total area of the post-tensioning steel: } A_{ps} = (564 + 1600) = 2164 \text{ mm}^2.
\]
\[ \frac{t_{pu}}{t_{cu \times b d}} = \left( \frac{1600 \times 2164}{40 \times 300 \times 600} \right) = 0.43 \]

\[ \frac{f_{pb}}{0.27 + t_{cu}} = 0.72 \text{ and } \]

\[ f_{pb} = \left( 0.72 \times 0.27 \times 1600 \right) = 1002 \text{ N/mm}^2 \]

\[ \left( \frac{f_{pb}}{f_{ct}} \right) = 0.75 \]

\[ x = 0.75 \times 500 \]

\[ d = 450 \text{ mm} \]

\[ M_u = f_{pb} A_p s (d - 0.45x) \]

\[ = 1002 \times 2164 \left( 600 - 0.45 \times 450 \right) \]

\[ = 862 \times 106 \text{ N} \cdot \text{mm} \]

\[ M_u = 862 \text{ kN} \cdot \text{m} \]
COMPOSITE STRUCTURAL MEMBERS

Precast prestressed members are used in conjunction with the concrete cast –in- situ, so that the members behave as monolithic unit under service loads is called composite construction.

High strength prestressed units are used in the tension zone and cast-in-situ of relatively lower compressive strength is used in the compression strength is used in the compression zone of the composite members.

Achieve compositeness between precast and cast in situ part

The composite action between the two components is achieved by roughening the surface of the prestressed unit on to which the concrete is cast in situ, thus giving a better frictional resistance or by stirrups protruding from the prestressed unit into the added concrete or by castellations on the surface of the prestressed unit adjoining the concrete which is cast in situ.

Composite construction of prestressed and in situ concrete
In a composite construction, precast prestressed members are used in conjunction with the concrete cast in situ, so that the members behave as monolithic unit under service loads. The high strength prestressed units are used in the tension zone while the concrete, which is the cast in situ of relatively lower compressive strength is used in the compression zone of the composite members.

In the case of composite members, deflections are computed by taking into account the different stages of loading as well as the differences in the modulus of elasticity of concrete in the precast prestressed unit and the in situ cast element.

**Unpropped construction**

If the precast units are not propped while placing the in situ concrete, stresses are developed in the unit due to the self weight of the member and the dead weight of the in situ concrete. This method of construction is referred to as unpropped construction.

**Forces considered in the calculation of deflection of prestressed concrete**

- Prestressing force
- Self weight of the beam
- Dead load of the concrete
- Live load acting on the concrete

**Role played by shear connectors in composite construction**

It is generally assumed that the natural bond at the interface contributes a part of the required shear resistance depending upon the strength of the in situ cast concrete and the roughness of the precast element. Any extra shear resistance over and above this should be provided by shear connectors.

**Advantages in using precast prestressed units**

- Saving in the cost of steel in a composite member compared with a reinforced or prestressed concrete member.
- Sizes of precast prestressed units can be reduced due to the effect of composite action.
- Low ratio of size of the precast unit to that of the whole composite member.
- Composite members are ideally suited for construction bridge decks without the disruption of normal traffic.

**Disadvantages of prestressed continuous beams**

- (i) Loss of prestress due to friction is more in long cables.
- (ii) Secondary stresses due to prestressing, creep, shrinkage, temperature & settlement of supports may induce very high stresses unless they are controlled.
- (iii) Cable positioned to cater for secondary moments are generally no suitable to provide the required ultimate moment under a given system of loads.
- (iv) The computation of collapse (or) ultimate load is influenced by the degree of redistribution of moments.

**Shrinkage and resultant stresses in composite member**

The magnitude of differential shrinkage is influenced by the composition of concrete and the environmental conditions to which the composite member is exposed.
absence of exact data, a general value of 100 micro strains is provided for computing shrinkage stresses.

**Assumptions made in stresses developed due to differential shrinkage**

- The shrinkage is uniform over the in situ part of the section.
- Effect of creep and increase in modulus of elasticity with age and the component of shrinkage, which is common to both the units are negligible.

**Loadings to be considered for computing deflection if the beam is propped section.**

- Prestress
- Self weight of the beam
- Self Weight of the in situ cast concrete

**Loadings to be considered for computing deflection if the beam is unpropped section.**

- Prestress
- Self weight of the beam
- Live load of the in situ cast concrete

**Advantages of statically indeterminate prestressed concrete structures?**

a. The bending moments are more evenly distributed between the centre of span and the supports of members.
   i. Reduction in the sizes of members results in lighter structure.
   ii. The ultimate load carrying capacity in higher than the statically determinate structure due to the redistribution of moment
   iii. Continuity of the members in framed by segmental construction using precast units connected by prestressed cables.

b. In continuous post tensioned guides, the curved cables can be suitably positioned to resist the span & support moments.

**Disadvantages of prestressed continuous beams**

(i) Loss of prestress due to friction is more in long cables.
(ii) Secondary stresses due to prestressing, creep, shrinkage, temperature & settlement of supports may induce very high stresses unless they are controlled.
(iii) Cable positioned to cater for secondary moments are generally no suitable to provide the required ultimate moment under a given system of loads.
(iv) The computation of collapse (or) ultimate load is influenced by the degree of redistribution of moments.

**Assumptions made for the analysis of secondary moment**

- The effect of change in length of members due to the prestressing force & external loading in negligible.
d. The cable friction is considered to be negligible so that the prestressing force is constant at all points of the cable.

**Composite construction is prestressed concrete**

In a composite construction, precast prestressed members are used in conjunction with the concrete cast in site. So that the members behave as a monolithic unit under service loads.

**Advantages of composite constructions**

a. Appreciable savings in the cost of steel in a composite member compared with a R.C.C. or prestressed concrete members.

b. Sizes of the precast prestressed units can be reduced due to the effect of composite action.

c. Composite constructions are indically suited for the constructions of bridge decks without disturb the normal traffic.

d. Efficient utilization of materials in a composite section results in reduced dead loads leading to overall economy.

**Methods of achieving continuity**

Continuity in precast prestressed concrete construction is achieved by using curved or straight cables which are continuous over several spans. It is also possible to develop continuity between two precast beams by using cap cables. Alternatively, short, straight tendons may be used over the supports to develop continuity between two precast prestressed beams.

**Methods of analysis of secondary moments.**

In addition to the basic assumptions such as, the elastic behavior of materials and linear strain distribution across the section, the following assumptions are generally made for the analysis of secondary moments in continuous prestressed concrete members.

1. The effect of change in the length of members due to the prestressing force and external loading is negligible.

2. The cable friction is considered to be negligible, so that the prestressing force is constant at all points of the cable.

There are several methods for analyzing statically indeterminate prestressed structures to compute the secondary moments that develop from prestressing the structure.

The most commonly used methods are based on the principles of,

1. Three moment theorem
2. Consistent deformation
3. Tendon reaction
Calculation of stresses.

Shear connectors

Effective bonding between the two parts of a composite beam may be developed by providing castellation in the precast unit or by projecting reinforcements from the precast unit is known as shear connectors.
16 marks:

Composition Construction of prestressed and in situ concrete.

A composite T-Beam is made up of a pretensioned rib 100mm wide and 200mm beam, and a cast in situ slab 400mm wide and 40mm thick having a modulus of elasticity of 23 kn/mm². If the differential shrinkage is $100 \times 10^{-6}$ units, determine the shrinkage stress developed in the precast and cast in situ units.

Solution:

Differential shrinkage, $E_{cs} = 100 \times 10^{-6}$

Area in situ concrete, $A_i = (400 \times 40) = 16000 \text{ mm}^2$.

Uniform tensile stress induced in the cast in-situ slab = $E_{cs} E_c$

$= (100 \times 10^{-6}) (23 \times 10^3) = 2.3 \text{ N/mm}^2$.

Force, $N_{sh} = E_{cs} E_c A_i = [(100 \times 10^{-6})(23 \times 10^3)(16 \times 10^3)]$

$= 44.8 \times 10^3$.

The centroid of the composite section is located 87 mm from the top fibre. Eccentricity from the centroid of composite section = $(87 - 20) = 67 \text{ mm}$.

Moment = $(44.8 \times 10^3) \times 67 = 3 \times 10^6 \text{ Nmm}$.

Second moduli for the various fibres:

Top fibre, $E_t = (2.25 \times 10^8) \text{ mm}^2$

Bottom fibre, $E_b = (1.23 \times 10^8) \text{ mm}^2$.

Junction, $E_j = (4.14 \times 10^4) \text{ mm}^2$.

Direct compressive stress = $\left(\frac{44.8 \times 10^3}{36 \times 10^3}\right) = 1.24 \text{ N/mm}^2$. 
Bending Stress:

\[
\text{Top fibre} = \frac{3 \times 10^6}{225 \times 10^4} = 1.33 \text{ N/mm}^2
\]

\[
\text{Bottom fibre} = \frac{3 \times 10^6}{128 \times 10^4} = 2.34 \text{ N/mm}^2
\]

\[
\text{Junction} = \frac{3 \times 10^6}{414 \times 10^4} = 0.72 \text{ N/mm}^2
\]

Differential shrinkage stresses:

a) In precast pretensioned beam (+ comp - tension)

At the top of beam = (1.24 + 0.72) = 1.96 N/mm²
At the bottom of beam = (1.24 - 2.34) = -1.10 N/mm²

b) In situ slab cast slab,

At the top of slab = (1.24 + 1.33 - 2.8) = -0.23 N/mm²
At the bottom of slab (junction) = (1.24 + 0.72 - 2.8) = -0.84 N/mm²

The resultant shrinkage stress distribution:

![Diagram of stress distribution]
A composite beam of rectangular section is made up of a pre-tensioned inverted T-beam having a slab thickness and width of 150 and 1200 mm respectively. The rib size is 150 mm by 850 mm. The cast in situ concrete has a thickness and width of 1500 mm with a modulus of elasticity of 30 kN/mm². If the differential shrinkage is 100 x 10⁻⁶ units, estimate the shrinkage stresses developed in the precast and cast in situ units.

Solution:

Differential shrinkage, \( \varepsilon_{cs} = 100 \times 10^{-6} \).

Area of in situ concrete, \( A_i = 872500 \text{ mm}^2 \).

Area of composite section, \( A_c = 1150 \times 10^3 \text{ mm}^2 \).

Uniform tensile stress induced in the cast in situ slab is

\[
\sigma = \varepsilon_{cs} E_{cs} = \left[ 100 \times 10^{-6} \right] \times 30 \times 10^3 \text{ N/mm}^2.
\]

For net force:

\[
\varepsilon_{cs} E_{cs} A_i = \left[ 100 \times 10^{-6} \right] \times 30 \times 10^3 \times 872500
\]

\[
= 2617500 \text{ N}.
\]

The centroid of the composite section is located 315 mm from the top fibre.

Eccentricity of the compressive force, \( N_k \), from the centroid of the composite section is

\[
N_k = (1575 - 500) = 1075 \text{ mm}.
\]

Moment = \( (2617500) \times 75 = (196.3 \times 10^6) \text{ N mm} \).

Second moment of area of the composite section = \( (26739 \times 10^6) \text{ mm}^4 \).
Section moduli for the various fibres,

pretensioned beam:

Top fibre = \((273 \times 10^6)\) mm\(^3\).
Bottom fibre = \((220 \times 10^6)\) mm\(^3\).

In-situ slab:

Top fibre = \((220 \times 10^6)\) mm\(^3\).
Bottom fibre = \((293 \times 10^6)\) mm\(^3\).

Direct compressive stress in the composite section:

\[ \left( \frac{2617500}{1150 \times 10^3} \right) = 2.276 \text{ N/mm}^2 \]

Bending stress:

Top fibre = \( \sqrt{\frac{196.3 \times 10^6}{220 \times 10^6}} \) = 0.892 N/mm\(^2\).

Bottom fibre = \( \sqrt{\frac{196.3 \times 10^6}{293 \times 10^6}} \) = 0.658 N/mm\(^2\).

Differential shrinkage stresses.

a) In the precast pretensioned beam (compressive):
   At the top of beam = \((2.276 + 0.658) = 2.934 \text{ N/mm}^2\)
   At the bottom of beam = \((2.276 - 0.892) = 1.384 \text{ N/mm}^2\)

b) In in situ cast slab:
   At the top of slab = \((2.276 + 0.892 - 3.30) = 0.163 \text{ N/mm}^2\).
   At the bottom of slab = \((2.276 - 0.658 - 3.00) = 1.322 \text{ N/mm}^2\).

\[ 0.163 \text{ (Tension)} \]

\[ +2.934 \text{ (Normal)} \]

1150 1800

1800 190
A composite T-girder of span 3m is made up of a pretensioned rib, 150 mm wide by 200 mm deep, with straight cable having area eccentricity 53.33 mm and carrying an initial force of 140 KN. The loss of prestress may be assumed to be 15 percent. Check the composite T-beam for the limit state of deflection if it supports an imposed load of 3.2 KN/m for a) unpropped construction, b) propped construction.
Assume a modulus of elasticity of 35 KN/mm² for both precast in situ cast elements.

Solution:

Self-weight of precast beam = 0.43 N/mm.
Self-weight of in-situ cast slab = 0.334 N/mm.
Imposed load on composite section = 3.2 N/mm.

$I$ for precast section = $(667 \times 10^5)$ mm⁴.
$I$ for composite section = $(1942 \times 10^5)$ mm⁴.
Modulus of elasticity, $E = (35 \times 10^3)$ N/mm².

Deflection due to prestress,

$$\left(\frac{P_e L^2}{8EI}\right) = \left[\frac{150 \times 10^3 \times 53.33 \times 5000^2}{8 \times 35 \times 10^2 \times 667 \times 10^5}\right] = 6.7 \text{ mm (upward)}.$$

Effective deflection after tures = $(0.85 \times 6.7) = 5.7 \text{ mm}$

Deflection due to self-weight of precast beam,

$$\left(\frac{5gL^4}{384EI}\right) = \left[\frac{5 \times 0.43 \times 5000^4}{384 \times 35 \times 10^2 \times 667 \times 10^5}\right] = 1.7 \text{ mm.}$$
Deflection of precast beam due to self-weight of cast in situ slab,

\[ = \left[ \frac{1.7 \times 0.334}{0.34} \right] = 1.24 \text{ mm.} \]

Deflection of composite beam due to live load

\[ = \left[ \frac{3.2 \times 5000}{8.4 \times 95 \times 10^5 \times 1943 \times 10^5} \right] = 0.47 \text{ mm.} \]

a) Unpropped construction:

Resultant deflection under service loads = \((-5.7 + 1.7 + 1.34 + 3.83)\)

= 1.14 mm.

b) Propped construction:

Resultant deflection under service loads = \((-5.7 + 1.7 + 0.40 + 3.83)\)

= 0.30 mm.

Due to \(f_s = 1843\), the maximum permissible deflection under service loads is limited to a value of \(\frac{2 \text{ span}}{250} = \frac{5000}{250}\)

= 20 mm.
The cross-section of a composite beam is of T-section having a pre-tensioned rib 80 mm wide and 240 mm deep and an inside cast slab, 350 mm wide and 80 mm thick. The pre-tensioned beam is reinforced with eight wires of 6 mm diameter with an ultimate tensile strength of 1600 N/mm², located 60 mm from soffit of beam. The comp. strength of concrete in the site cast and precast element is 20 x 40. If adequate reinforcement are provided, estimate the flexural strength of composite section.

Solution.

\[ A_p = 8 \times \frac{\pi}{4} \times 5^2 = 160 \text{ mm}^2 \]

\[ f_{ck} = 20 \text{ N/mm}^2 \]

\[ f_p = 1600 \text{ N/mm}^2, \quad d = 350 \text{ mm} \]

Effective reinforcement ratio:

\[ \frac{f_p}{A_p} = \frac{1600 \times 160}{20 \times 350 \times 10^4} = 0.152 \]

Referring to table 10.2, \( f_k = 1343 - 1980 \)

\[ \frac{f_p}{f_k} = 0.37 \]

\[ f_p = 0.37 \times 1600 = 1392 \text{ N/mm}^2 \]

\[ f_k = 0.326 \]

\[ X_u = 0.326 \times 240 = 78 \text{ mm} \]

Flexural composite section:

\[ M_u = f_p A_p (d - 0.42 X_u) \]

\[ = 1392 \times 160 \times \left[ \frac{240 - 0.42 \times 78}{10^6} \right] \]

\[ M_u = 4615 \text{ kNm} \]
UNIT 5 - MISCELLANEOUS STRUCTURES

Prestressed tension member:

Prestressed tension member have the following three specific behaviors’,

1. the member can be considered as essentially made of concrete which is put under uniform compression, so that it can carry tension produced by internal pressure or external loads, if the concrete has no cracked, it is able to carry a total tensile force equal to the total effective pre compression plus the tensile capacity of the concrete itself.

2. the member can be considered as essentially made of high tensile steel which is prolonged to reduce the deflection under load, from the view point, the ultimate strength of the member is dependent upon the tensile strength of the steel, but the usable strength is often limited by excessive elongation of the steel following the cracking of the concrete.

3. the member can be considered as a combined steel and concrete member whose strains and stresses before cracking can be evaluated, assuming elastic behavior and taking into account the effect of shrinkage and creep.

The design criteria for prestressed concrete tanks

- It is to resist the hoop tension and moments developed are based on the considerations of desirable load factors against cracking and collapse.
- It is desirable to have at least a minimum load factor of 1.2 against cracking and 2 against ultimate collapse as per IS code.
- It is desirable to have at least a minimum load factor of 1.25 against cracking and 2.5 against ultimate collapse as per BS code.
- The principal compressive stress in concrete should not exceed one-third of the characteristic cube strength.
- When the tank is full, there should be a residual compressive stress of at least 0.7 N/mm². When the tank is empty, the allowable tensile stress at any point is limited to 1 N/mm². The maximum flexural stress in the tank walls should be assumed to be numerically equal to 0.3 times the hoop compression.

Circular prestressing.

When the prestressed members are curved in the direction of prestressing, the prestressing is called circular prestressing. For example, circumferential prestressing in pipes, tanks, silos, containment structures and similar structures is a type of circular prestressing.

The tanks classified based on the joint.

- Tank wall with fixed base.
- Tank wall with hinged base.
- Tank wall with sliding base.
The different types of joints used between the slabs of prestressed concrete tanks
- Movement joint
- Expansion joint
- Construction Joint
- Temporary Open Joints.

The stress induced in concrete due to circular prestressing
The circumferential hoop compression stress is induced in concrete by prestressing to counterbalance the hoop tension developed due to the internal fluid pressure.

The advantages of prestressing water tanks
- Water storage tanks of large capacity are invariably made of prestressed concrete.
- Square tanks are used for storage in congested urban and industrial sites where land space is a major constraint.
- This shape is considerable reduction in the thickness of concrete shell.
- The efficiency of the shell action of the concrete is combined with the prestressing at the edges.

The functions of water stopper (water bar) in water tank construction
- The base slab is subdivided by joints which are sealed by water stops.
- The reinforcement in the slab should be well distributed to control the cracking of the slab due to shrinkage and temperature.

The design criteria for prestressed concrete pipes
- Circumferential prestressing, winding with or without longitudinal prestressing. Handling stresses with or without longitudinal prestressing.
- Condition in which a pipe is supported by saddles at extreme points with full water load but zero hydrostatic pressure.
- Full working pressure conforming to the limit state of serviceability. The first crack stage corresponding to the limit state of local damage.

Prestressed cylinder and non-cylinder pipe.

Prestressed cylinder pipe:
- It is developed by the Lock Joint Company.
- A welded cylinder of 16 gauge steel is lined with concrete inside and steel pipe wrapped with a highly stressed wire.
- Tubular fasteners are used for the splices and for end fixing of the wire and pipe is finished with a coating of rich mortar.
- It is suitable upto 1.2 m diameter.

Prestressed non-cylinder pipe:
- It is developed by Lewiston Pipe Corporation.
- At first concrete is cast over a tensioned longitudinal reinforcement.
- A concrete pipes after curing are circumferentially stressed by means of a spiral wire.

The types of prestressed concrete pipes construction
Monolyte construction

Two stage construction.

Non-cylinder pipes:
The design principles are used for determining the minimum thickness of concrete required and the pitch of circumferential wire winding on the pipe.

Cylinder pipes:
The design principles of cylinder pipes are similar to those of the non-cylinder pipes except that the required thickness of concrete is computed by considering the equivalent area of the light gauge steel pipe embedded in the concrete.

The advantages of prestressed concrete piles
☑ High load and moment carrying capacity.
☑ Standardization in design for mass production.
☑ Excellent durability under adverse environmental conditions. Crack free characteristics under handling and driving.
☑ Resistance to tensile loads due to uplift. Combined load moment capacity.

Prestressed concrete poles:

The effect of prestressing force in concrete poles.
It should be reduced in proportion to the cross section by the techniques of debonding or dead ending or looping some of the tendons at mid height.

The types of loadings that act on prestressed concrete poles.
Bending due to wind load on the cable and on the exposed face. Combined bending and torsion due to eccentric snapping of wires. Maximum torsion due to skew snapping of wires. Bending due to failure of all the wires on one side of the pole. Handling and erection stresses.

The importance of shrinkage in composite construction
The time dependent behavior of composite prestressed concrete beams depends upon the presence of differential shrinkage and creep of the concretes of web and deck, in addition to other parameters, such as relaxation of steel, presence of untensioned steel, and compression steel etc.

The advantages of partially prestressed concrete poles
☑ Resistance to corrosion in humid and temperature climate and to erosion in desert areas.
☑ Easy handling due to less weight than other poles.
☑ Easily installed in drilled holes in ground with or without concrete fill.
☑ Lighter because of reduced cross section when compared with reinforced concrete poles.
☑ Fire resisting, particularly grassing and pushing fire near ground line.

The advantages of prestressed concrete over R.C.C concrete.
The use of high strength concrete and steel in prestressed members results in lighter and slender members than is possible with reinforced concrete.

The effectiveness of carrying external loads is only by the section above the neutral axis is reinforced concrete but the entire cross section is effective is prestressed concrete.

**Partial prestressing.**

Under the working load, if the cross section is subjected to no tension after prestressing then it is known as fully prestressed. Under working loads even after the pretsress is apply. If there is some tension. It is known as partial prestressing. Normally the tension portion is reinforced with mild steel reinforcement. This untensioned reinforcement is required so as to resist differential shrinkage temperature effects and handling stresses.

**Limited prestressing**

It leads to a reduction in the cost of stressing, sheathing and grouting. The use of high yield strength deformed bars is generally believed to offer better crack control and high ultimate strength.

**Objectives of partial prestressing:**

- Better distribution of stress
- Reinforce regions of peak moment
- Control on excessive camber
- Control of crack width
- Increase of ductility and rotation capacity
- Limit state collapse condition

**Methods of achieving partial prestressing**

Partial prestress may be obtained by any of the following measures,

- **By using less high tensile steel for prestressing,** this will save steel, but will also decrease the ultimate strength, which is almost directly proportional to the amount of steel.
✓ By using the same amount of high tensile steel, but leaving some non-prestressed. This will save some tensioning and anchorage, and may increase resilience at the sacrifice of earlier cracking and slightly smaller ultimate strength.

✓ By using same amount of steel, but tensioning them to a lower level. The effects of this are similar to those, but no end anchorage are saved.

✓ By using less prestressed steel and adding some mild steel for reinforcing. This will give the desired ultimate strength and will result in greater resilience at the expense of earlier cracking.

**Merits**

✓ Camber of bridge deck is better controlled
✓ Savings in the amount of prestressing steel.
✓ Greater resilience in the structure is possible.
✓ Economical utilization of mild steel.

**Demerits**

✓ Earlier appearance of cracks
✓ Greater deflections under overloads
✓ Higher principal tensile stress under working loads.
✓ Slight decrease in ultimate flexural strength for the same amount of steel.

**The advantages of partial prestressing.**

✓ Limited tensile stresses are permitted in concrete under service loads with controls on the maximum width of cracks and depending upon the type of prestressing and environmental condition.

✓ Untensioned reinforcement is required in the cross-section of a prestressed member for various reasons, such as to resist the differential shrinkage, temperature effects and handling stresses.
16 marks

Design a non-cylinder prestressed concrete pipe of 600 mm internal diameter to withstand a working
hydraulic pressure of 1.05 N/mm², using 2.5 mm high
tensile wire stressed to 1800 N/mm² at transfer. Permissible
maximum and minimum stress in concrete at transfer
and service loads are 14 and 0.7 N/mm². The lots of
ratio is 0.8. Calculate also the test pressure required
to produce a tensile stress of 0.7 N/mm² in concrete
when applied immediately after tensioning and also
the winding stress in steel if \( E_s = 210 \text{ kN/mm}^2 \) and
\( E_c = 35 \text{ kN/mm}^2 \).

**Solution:**

1. \[ l > \frac{N_d}{f_{ctk} + f_{min.w}} = \frac{1.05 (600/2)}{0.8 \times 14 - 0.7} > 30 \text{ mm} \]

   For a 30 mm thick concrete pipe, the actual \( f_c \) in
   concrete \( f_c = 14 \text{ N/mm}^2 \).

   The number of turns of 2.5 mm wire stressed
   to 1800 N/mm² per metre length of the pipe is given by,

2. \[ n = \frac{45000 \times f_c}{\pi D^2 f_{st}} = \frac{45000 \times 30 \times 14}{\pi \times 2.5^2 \times 1800} = 26 \text{ turns/m} \]

   \[ \text{pitch} = \frac{1500}{26} = 57.63 \text{ mm} \]

   \[ \Rightarrow r = 1 \]

3. \[ f_c = \frac{N_w - f_{min.w}}{2n \pi D} \]

   \[ N_w = \frac{25}{D} (f_c - f_{min.w}) = \frac{2 \times 35 [14 - (0.7)]}{600} = 1.47 \text{ N/mm}^2 \]

4. \[ f_{si} = \text{Winding stress in steel} = (1 + x_c p) f_c \]

   \[ x_c = 6 \text{ and } p = \frac{f_c}{f_{st}} = \frac{14}{1800} = 0.007778 \]

   \[ f_{si} = (1 + 6 \times 0.007778) 1800 = 1084 \text{ N/mm}^2 \]
A non-cylinder prestressed concrete pipe of 1.6 m diameter with a core thickness of 100 mm is required to withstand a working pressure of 11.4 MPa. Determine the pitch of a 5 mm diameter wire winding. If the high-tensile initial stress in the wire is limited to 1020 MPa, the permissible maximum and minimum stresses in concrete are 12.3 MPa (compressive) and zero (tensile). The loss ratio is 0.5. If the direct tensile strength of concrete is 2.11 MPa, estimate the load factor against cracking.

Solution:

Minimum thickness of pipe required

\[
\begin{align*}
L & > \frac{1.0 (1600 / 2)}{0.5 \times 12 - 0} \\
& > 84 \text{ mm}
\end{align*}
\]

Thickness provided \( t = 100 \text{ mm} \)

\[
\begin{align*}
\sigma_c &= \frac{1 \times 600}{2 \times 0.5 \times 100} = 10.2 \text{ MPa}.
\end{align*}
\]

No. of wires/m, \( n = \frac{4 \times 100 \times 100 \times 10}{\pi \times 5^2 \times 100} = 51 \text{ turns/m} \)

Pitch of winding \( = \frac{1050}{51} = 20.6 \text{ mm} \).

Hoop tension due to fluid pressure \( = \frac{1 \times 1600}{2 \times 100} = 8 \text{ MPa} \).

Hoop compression due to prestress \( = 10 \text{ MPa} \).

Resultant compressive stress in concrete \( = 10 - 3 = 7 \text{ MPa} \).

Tensile strength of concrete \( = 2.11 \text{ MPa} \).

Additional fluid pressure required to develop a tensile stress of 4 MPa in concrete is given by:

\[
\begin{align*}
\sigma &= \frac{2 \times 1500 \times 4}{1000} = 0.6 \text{ MPa}.
\end{align*}
\]

Cracking fluid pressure \( = 1 + 0.5 = 1.5 \text{ MPa} \).

Working pressure \( = 1 \text{ MPa} \).

Load factor against cracking \( = 1.5 \times 1 = 1.5 \).
A non-circular precast concrete pipe of internal diameter with a core 1800 mm and thickness of concrete shell 75 mm is required to convey water at a working pressure of 1.5 N/mm². The length of each pipe is 6 m. The maximum direct compressive stress in concrete are 15 and 2 N/mm². The loss ratio is 0.8.

a) Design the circumferential wire winding using 5 mm diameter wires stressed to 1500 N/mm².

b) Design the longitudinal prestressing using 7 mm wires tensioned to 10,000 N/mm². The maximum permissible tensile stress under the critical transient loading should not exceed 0.8 $f_{ci}$ where $f_{ci}$ is the cube strength of concrete at transfer = 40 N/mm².

c) Check the safety against longitudinal stresses that develop, considering the pipe as a hollow circular beam as per IS: 743 provisions.

**Solution:**

$D = 1800$ mm, $t_{ct} = 15$ N/mm², $t_{min. w} = 2$ N/mm², $f_s = 1500$ N/mm², $W = 1.5$ N/mm², $t = 75$ mm, $L = 6$ m.

a) **Circumferential Wire Winding:**

Compressive stress in concrete,

$$t_c = \frac{N_{ol}}{2t} + \frac{t_{min. w}}{n} = 15 \left( \frac{1000}{2} \right) + \frac{2}{0.8 \times 75} = 15 \text{ N/mm}^2.$$  

Number of turns,

$$n = \frac{4000 \times t_c}{1000 \times 75} = \frac{4000 \times 75 \times 15}{1000 \times 5^2 \times 1000} = 57 \text{ turns/m}.$$  

Pitch of winding = $\frac{1000}{57} = 17.5\text{ mm}.$
b) **Longitudinal prestressing:**

Critical transient stresses at spigot end

\[ = 0.6 \times \text{hoop stress} \]

\[ = 0.6 \times 15 = 9 \text{ N/mm}^2 \]

Maximum permissible tensile stresses

\[ = 0.3 \frac{f_{ci}}{f_{yd}} \]

\[ = 0.3 \frac{540}{5} = 5 \text{ N/mm}^2 \]

Here, the tensile stress of 9 - 5 = 4 N/mm² should be counterbalanced by longitudinal prestressing.

Cross-sectional area of the pipe

\[ = \pi \times 1.075 \times 0.075 \text{ m}^2 \]

If \( P \) is the longitudinal prestressing force required, then,

\[ P = \frac{\pi \times 1.075 \times 0.075 \times 10^6 \times 4 \text{ kN}}{10^3} \]

\[ = 1013 \text{ kN} \]

Using 7mm wires stressed to 1000 N/mm².

Force in each wire = 33.5 kN

Number of wires = \( \frac{1013}{33.5} = 30 \)

C) **Check for flexural stresses as per IS: 734.**

Considering the pipe as a beam of hollow circular section over a span of 6m.

Three times self-weight

\[ = 3 \pi \times 1.075 \times 0.075 \times 29 \]

\[ = 18.30 \text{ kN/m} \]

Weight of water

\[ = \frac{1}{4} \times (\pi \times 120 \times 10) = 7.90 \text{ kN/m} \]

Total load on pipe

\[ = 26.20 \text{ kN/m} \]

Maximum bending moment

\[ = \frac{26.2 \times 6^2}{8} = 118 \text{ kN.m} \]

Second moment of area

\[ I = \frac{\pi (1.5^4 - 1.4^4)}{64} = 0.0365 \text{ m}^4 \]
Flexure tensile stress = \( \frac{118 \times 10^6 \times 575}{0.0365 \times 10^{12}} \)
= 1.38 N/mm² (Tension)

Longitudinal prestress = 4 N/mm².

Resultant stress in concrete = 4 - 1.38
= 2.12 N/mm² (comp)

The resultant stress being compressive, the pipe is safe against cracking.

Explain the design procedure of pre-cylindrical water tank.

Step 1: The dimensions of the tank based on storage capacity required, space available etc., may be first arrived at.

Step 2: Then the system and type of prestressing to be adopted may be fixed.

Step 2: Effective prestressing force that could be achieved with one cable pulling is evaluated.

Step 4: Hoop tensile forces generated due to water load is evaluated. The ring force diagram on a vertical section of the tank is then plotted. The spacing of wires based on the ring force diagram and effective prestress that could be transferred by one cable is then decided.
Step 5: The flexural moment generated due to cable tension is evaluated.

Step 6: The moment due to water pressure is evaluated.

Step 7: Ring forces at different levels moment due to water pressure and prestress are tabulated and design moments are evaluated.

Step 8: The vertical prestressing required is calculated or wall is to be designed for vertical moments as RCC.

Step 9: Plasters for anchoring the cables have to be designed.

A prestressed concrete circular cylindrical tank is required to store 24,500 million liters of water. The permissible compressive stress in concrete at transfer should not be less than exceed 13 N/mm² and the minimum compressive stress under working pressure should not be less than 1 N/mm². The loss ratio is 0.75. High tensile steel wires of 7 mm diameter with an initial stress of 1000 N/mm², are available for winding round the tank. Freyssinet cables of 12 wires of 3 mm diameter which are stressed at 1200 N/mm², are available for vertical prestressing. The cube strength of cement is 40 N/mm² design the tank walls supported on elastomeric pads. Assume the coefficient of friction as 0.5.

Volume of tank = 24,500 x 10⁶ litres.
Solution:

Assuming the diameter of tank as 50 m, height of storage = 12.5 m.

Form code, provision the thickness of the tank wall at the base is taken as 450 mm, which gradually reduces to 200 mm towards the top of the tank.

Hydrostatic pressure, \( W_H = WH = (10 \times 12.5) = 125 \text{ kN/m}^2 \) = \( 0.125 \text{ kN/mm}^2 \).

Maximum ring tension, \( N_t = (10 \times 12.5 \times 125) = 15625 \text{ kN/m}^2 \).

Self-weight of the wall = \( (12.5 \times 0.3 \times 1 \times 24) = 90 \text{ kN/m} \).

Frictional force at base, \( N_b = (0.5 \times 70) = 45 \text{ kN/m} \).

Minimum wall thickness at base = \( \frac{3125}{(0.75 \times 360) - 1} \) = 260 mm

Net thickness available is \( (400 - 40) = 360 \text{ mm} \).

Circumferential pressure,

\[
\sigma_c = \frac{3125}{0.75 \times 360} + 1 = 18 \text{ kN/mm}^2.
\]

Spacing of circumferential wire winding is

\[
S = \frac{2 \times 3125 \times 1500 \times 8.35}{0.125 \times 13 \times 50 \times 10^2 \times 260} = 3.2 \text{ mm}
\]

No. of wires/meter = 120

Ring tension at 0.75 m from top = \( (10 \times 0.75 \times 25) = 188 \text{ kN/m} \).

Thickness at top = 280 mm

Net thickness = \( (280 - 40) = 160 \text{ mm} \).
\[ t_e = \frac{138}{0.75 \times 160} + \frac{1}{0.75} = 2.91 \, \text{N/mm}^2. \]

\[ S = \frac{2 \times 138 \times 1600 \times 98.5}{0.125 \times 0.91 \times 50 \times 10^2 \times 160} = 50 \, \text{mm} \]

Number of wires at top/metre = 20

Maximum radial pressure due to prestress at transfer,

\[ W_L = \frac{20 \times 1000 \times 98.5}{5.8 \times 50 \times 10^2} = 0.186 \, \text{N/mm}^2. \]

Maximum vertical moment due to working pressure,

\[ M_w = 0.247 \, \text{N/m} \sqrt{E} \]
\[ = 0.247 \times 45 \times \sqrt{25 \times 0.4} \]
\[ = 35.5 \, \text{KN/m}. \]
\[ = 35500 \, \text{N/mm/m}. \]

Maximum vertical moment due to prestress is

\[ M_T = 98800 \left( \frac{0.186}{0.125} \right) = 53000 \, \text{N mm/mm} \]
\[ = 53 \times 10^6 \, \text{N mm/m}. \]

Considering one metre length of tank along the circumference, the section modulus is

\[ Z = \frac{1000 \times 450^2}{6} = 266 \times 10^6 \, \text{mm}^3. \]

The vertical prestress required,

\[ t_e = \left[ \frac{1}{0.75} + \frac{53 \times 10^6}{266 \times 10^6} \right] = 3.33 \, \text{N/mm}^2. \]
As per the IS code, the minimum vertical prestress required to counteract the bending stresses is 

\[ \sigma = (0.3 \times 12) = 3.6 \text{ kN/m}^2 \]

\[ \text{vertical prestressing force} = \frac{(3.6 \times 1500 \times 400)}{1500} = 1560 \text{ kN} \]

Spacing of vertical cables = \( \frac{1500 \times 720}{1560} = 460 \text{ mm} \)

Ultimate tensile force in wires at base of tank:

\[ \text{Load factor against collapse} = \frac{6900}{3125} = 2.2 \]

Cracking load = \( \frac{(1500 \times 400) \times (0.75 \times 13 \times 1.7)}{(100)} = 4580 \text{ kN} \)

Factor of safety against cracking = \( \frac{4580}{3125} = 1.47 \)

Nominal reinforcements of 0.2 per cent of the cross-section in the circumferential and vertical direction are well distributed on each face.